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14. ABSTRACT
We propose development of distributed estimation and control algorithms for design of reliable guidance, navigation and control systems for autonomous vehicles with the use of sensor networks that can enable more precise navigation laws, lower cost and increased reliability. The technical focus is the integration of estimation and control algorithms for multiple vehicles that would lead to guaranteed performance bounds in uncertain, and possibly occluded, environments in the presence of communication losses and network failures.

15. SUBJECT TERMS
Uncertain system, Multivehicle autonomous systems, Sensor networks, Decentralized estimation, Adaptive navigation and guidance, Guaranteed performance

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Report Title

Decentralized Estimation and Vision-based Guidance of Fast Autonomous Systems with Guaranteed Performance in Uncertain Environments

ABSTRACT

We propose development of distributed estimation and control algorithms for design of reliable guidance, navigation and control systems for autonomous vehicles with the use of sensor networks that can enable more precise navigation laws, lower cost and increased reliability. The technical focus is the integration of estimation and control algorithms for multiple vehicles that would lead to guaranteed performance bounds in uncertain, and possibly occluded, environments in the presence of communication losses and network failures.

Enter List of papers submitted or published that acknowledge ARO support from the start of the project to the date of this printing. List the papers, including journal references, in the following categories:

(a) Papers published in peer-reviewed journals (N/A for none)

Received		Paper
04/22/2013	29.00	Enric Xargay, Vladimir Dobrokhodov, Isaac Kaminer, Antonio Pascoal, Naira Hovakimyan, Chengyu Cao. Time-Critical Cooperative Control of Multiple Autonomous Vehicles: Robust Distributed Strategies for Path-Following Control and Time-Coordination over Dynamic Communications Networks, IEEE Control Systems, (10 2012): 49. doi: 10.1109/MCS.2012.2205477
04/22/2013	28.00	E. Xargay, I. Kaminer, A. Pascoal, N. Hovakimyan, V. Dobrokhodov, V. Cichella, A. P. Aguiar, R. Ghabcheloo. Time-Critical Cooperative Path Following of Multiple Unmanned Aerial Vehicles over Time-Varying Networks, Journal of Guidance, Control, and Dynamics, (03 2013): 499. doi:
08/03/2011	6.00	Chengyu Cao, Naira Hovakimyan, Vladimir Dobrokhodov, Isaac Kaminer, Lili Ma. Adaptive Vision-Based Guidance Law with Guaranteed Performance Bounds, Journal of Guidance, Control, and Dynamics, (5 2010): 0. doi: 10.2514/1.46287
08/03/2011	13.00	Isaac Kaminer, Antonio Pascoal, Enric Xargay, Naira Hovakimyan, Chengyu Cao, Vladimir Dobrokhodov. Path Following for Unmanned Aerial Vehicles Using L1 Adaptive Augmentation of Commercial Autopilots, Journal of Guidance, Control, and Dynamics, (3 2010): 0. doi: 10.2514/1.42056
08/22/2011	17.00	L. Ma, C. Cao, N. Hovakimyan, C. Woolsey, W. E. Dixon. Fast estimation for range identification in the presence of unknown motion parameters, IMA Journal of Applied Mathematics, (02 2010): 165. doi: 10.1093/imamat/hxq008
TOTAL:		**5**

Number of Papers published in peer-reviewed journals:

(b) Papers published in non-peer-reviewed journals (N/A for none)

Received Paper

TOTAL:

Number of Papers published in non peer-reviewed journals:

(c) Presentations

Number of Presentations: 0.00

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Received Paper

TOTAL:

Number of Non Peer-Reviewed Conference Proceeding publications (other than abstracts):

Peer-Reviewed Conference Proceeding publications (other than abstracts):

Received		Paper
08/03/2011	5.00	Vladimir Dobrokhodov, Isaac Kaminer, Enric Xargay, Zhiyuan Li , Naira Hovakimyan. On Coordinated Road Search using Time-Coordinated Path Following of Multiple UAVs, AIAA Guidance, Navigation, and Control Conference. 2010/08/02 01:00:00, . : ,
08/03/2011	12.00	V. Cichella, I. Kaminer, V. Dobrokhodov , E. Xargay, N. Hovakimyan, A. Pascoal. Geometric 3D Path-Following Control for a Fixed-Wing UAV on SO(3), AIAA Guidance, Navigation, and Control Conference. 2011/08/08 01:00:00, . : ,
08/03/2011	11.00	Naira Hovakimyan, Dusan Stipanovic, Zhiyuan Li. Distributed Target Tracking and Collision Avoidance using Multiple Nonholonomic Robots with Uncertain Dynamics, AIAA Guidance, Navigation, and Control Conference. 2011/08/08 01:00:00, . : ,
08/03/2011	9.00	Zhiyuan Li, Naira Hovakimyan, Dusan Stipanovic. Distributed Multi-Agent Tracking and Estimation with Uncertain Agent Dynamics, American Control Conference. 2011/06/29 01:00:00, . : ,
08/17/2012	21.00	Zhiyuan Li, Naira Hovakimyan. L1 Adaptive Controller for MIMO system with Unmatched Uncertainties using Modi?ed Piecewise Constant Adaptation Law, IEEE 51st Conference on Decision and Control . 2012/12/10 01:00:00, . : ,
08/17/2012	19.00	Ronald Choe, Enric Xargay , Naira Hovakimyan, Isaac Kaminer. Convergence of a PI Coordination Protocol in Networks with Switching Topology and Quantized Measurements, IEEE 51st Conference on Decision and Control. 2012/12/10 01:00:00, . : ,
08/19/2011	15.00	Ronald Choe and Naira Hovakimyan. Perching Maneuver for an MAV Augmented with anL1 Adaptive Controller, AIAA Guidance, Navigation, and Control Conference. 2011/08/08 01:00:00, . : ,
08/20/2012	22.00	Xiaofeng Wang, Naira Hovakimyan. A Decoupled Design in Distributed Control of Uncertain Networked Control Systems, American Control Conference. 2012/07/27 01:00:00, . : ,
08/20/2012	20.00	Isaac Kaminer, Enric Xargay, Vladimir Dobrokhodov, Naira Hovakimyan, Venanzio Cichella, A. Pedro Aguiar, Antonio M. Pascoal. A Lyapunov-based approach for Time-Coordinated 3D Path-Following of multiple Quadrotors in SO(3), IEEE 51st Conference on Decision and Control . 2012/12/10 01:00:00, . : ,
08/22/2011	18.00	Kwang Ki Kim , Evgeny Kharisov , Naira Hovakimyan. Limiting Behavior of L1 Adaptive Controllers , AIAA Guidance, Navigation, and Control Conference. 2011/08/08 01:00:00, . : ,
08/23/2012	23.00	Justin Vanness, Evgeny Kharisov, Naira Hovakimyan. Generalization of Proportional Adaptation Law for L1 Adaptive Controller, American Control Conference. 2012/06/27 01:00:00, . : ,
TOTAL:	**11**	

Number of Peer-Reviewed Conference Proceeding publications (other than abstracts):

(d) Manuscripts

Received Paper

08/03/2011 14.00 Enric Xargay, Vladimir Dobrokhodov, Isaac Kaminer, Antonio Pascoal, Naira Hovakimyan. Time-Coordinated Path Following of Multiple Heterogeneous Vehicles over Time-Varying Networks, IEEE Control System Magazine (08 2011)

08/22/2011 16.00 Enric Xargay, Naira Hovakimyan, Vladimir Dobrokhodov, Isaac Kaminer, Chengyu Cao, Irene Gregory. L1 Adaptive Control in Flight, (submitted) (08 2011)

08/24/2012 25.00 This paper addresses the problem of steering a fleet of UAVs along desired 3-D paths, while meeting stringent spatial and temporal constraints. A representative example is the, challenging mission scenario where the UAVs are tasked to cooperatively execute collision-free maneuvers and arrive at their final destinations at the same time, or at different times, so as to meet a desired inter-vehicle schedule. In the proposed framework, the UAVs, are assigned nominal spatial paths and speed profiles along those, and then the vehicles, are requested to execute cooperative path following, rather than “open-loop” trajectory-tracking maneuvers. This strategy yields robust behavior against external disturbances by, allowing the UAVs to negotiate their speeds along the paths in response to information, exchanged over the dynamic inter-vehicle communications network., The proposed approach addresses explicitly the situation where each vehicle transmits, coordination-relevant information to only a subset of the other vehicles, as determined by, the communications topology. Furthermore, the paper considers the case where the graph, that captures the underlying communications topology is disconnected during some interval, of time or even fails to be connected at all times. Conditions are given under which the, complete time-critical cooperative path-following closed-loop system is stable and yields, convergence of a conveniently defined cooperation error to a neighborhood of the origin., Flight test results of a coordinated road search mission demonstrate the efficacy of the, multi-UAV cooperative control framework developed in the paper.. Time-Critical Cooperative Path Followingof Multiple UAVs over Time-Varying Networks, Journal of Guidance, Control, and Dynamics (09 2011)

08/28/2012 26.00 Enric Xargay, Vladimir Dobrokhodov, Isaac Kaminer , Antonio Pascoal, Naira Hovakimyan, Chengyu Cao. Time-Critical Cooperative Control of Multiple Autonomous Vehicles, IEEE Control System Magazine (09 2011)

TOTAL: 4

Number of Manuscripts:

Books

Received Paper

TOTAL:

Patents Submitted

Patents Awarded

Awards

Naira Hovakimyan was selected to receive the 2011 American Institute of Aeronautics and Astronautics (AIAA) Mechanics and Control of Flight Award

Enric Xargay received Roger A. Strehlow Memorial Award by the Aerospace Engineering Department of UIUC in May 2011.

Dr. Hovakimyan gave a keynote at ICNPAA in Vienna and was honored with a technical achievement award for her contributions to
ICNPAA on July 27, 2012. http://naira.mechse.illinois.edu/2012/07/27/icnpaa/

Prof. Hovakimyan has been recognized as University Scholar in UIUC on September 28, 2011. The program recognizes excellence while
helping to identify and retain the university's most talented teachers, scholars and researchers.
http://naira.mechse.illinois.edu/2011/09/28/prof-hovakimyan-named-university-scholar/

Graduate Students

NAME	PERCENT_SUPPORTED	Discipline
Zhiyuan Li	0.50	
FTE Equivalent:	**0.50**	
Total Number:	**1**	

Names of Post Doctorates

NAME	PERCENT_SUPPORTED
FTE Equivalent:	
Total Number:	

Names of Faculty Supported

NAME	PERCENT_SUPPORTED	National Academy Member
Naira Hovakimyan	0.10	
Dusan Stipanovic	0.10	
FTE Equivalent:	**0.20**	
Total Number:	**2**	

Names of Under Graduate students supported

NAME	PERCENT_SUPPORTED
FTE Equivalent:	
Total Number:	

Student Metrics

This section only applies to graduating undergraduates supported by this agreement in this reporting period

The number of undergraduates funded by this agreement who graduated during this period: 0.00

The number of undergraduates funded by this agreement who graduated during this period with a degree in science, mathematics, engineering, or technology fields:...... 0.00

The number of undergraduates funded by your agreement who graduated during this period and will continue to pursue a graduate or Ph.D. degree in science, mathematics, engineering, or technology fields:...... 0.00

Number of graduating undergraduates who achieved a 3.5 GPA to 4.0 (4.0 max scale): 0.00

Number of graduating undergraduates funded by a DoD funded Center of Excellence grant for Education, Research and Engineering: 0.00

The number of undergraduates funded by your agreement who graduated during this period and intend to work for the Department of Defense 0.00

The number of undergraduates funded by your agreement who graduated during this period and will receive scholarships or fellowships for further studies in science, mathematics, engineering or technology fields: 0.00

Names of Personnel receiving masters degrees

NAME

Total Number:

Names of personnel receiving PHDs

NAME

Zhiyuan Li

Total Number: 1

Names of other research staff

NAME	PERCENT_SUPPORTED

FTE Equivalent:

Total Number:

Sub Contractors (DD882)

Inventions (DD882)

Scientific Progress

Decentralized estimation and control algorithms for design of reliable guidance, navigation and control systems for autonomous vehicles with the use of sensor networks that can enable more precise navigation laws, lower cost or smaller future systems, and increased reliability, are the objectives of this research. The technical focus is the integration of estimation and control algorithms for multiple vehicles that would lead to guaranteed performance bounds in uncertain, and possibly occluded, environments in the presence of communication losses and network failures.

During the contract period we explored design of distributed cooperative control laws for multi-agent systems that take the uncertainties in the agent dynamics into consideration.

1. We proposed a cascaded control structure for multi-agent coordination in the presence of uncertain agent dynamics and disturbances, which resolved the coupling between the communication topology and the system dynamics. We implemented and tested the proposed algorithms on the UIUC multi-robot testbed.

2. We extended the cascaded control structure for state-dependent (relative-position-induced) network topology to enable a decoupled design for distributed control with event-triggered sampling. The decoupled architecture helps to leverage tools from network theory and robust control to enable the operation of the large-scale system with guaranteed performance bounds and robustness margins.

3. We developed a PI consensus algorithm for a multi-agent system with disturbances in each agent's dynamics operating over time-varying communication topologies and with quantized feedback.

4. We designed time-coordinated 3D path-following algorithms for multiple quadrotors and flight-tested the algorithms at the Naval Postgraduate School.

5. L1 adaptive controller is the core technology that enables distributed control design for multi-agent systems with uncertainties and guaranteed performance. We have made several modifications to the standard L1 adaptive controller algorithm to relax the CPU requirements, making it more suitable for systems with limited computation and sensing capabilities.

6. Throughout our research, we have been considering coordination under a very realistic communication setup, i.e., we have assumed distance-based network or network with a integral type of connectivity (which is a much weaker assumption than the point-wise connectivity), and shown the robustness of our coordination algorithms. We start to consider stochastic network which is in nature closer to the physical communication link used in practice.

Technology Transfer

Robust Architectures for Multi-Agent Systems

Army Research Office Final Report

Naira Hovakimyan, Dusan Stipanovic

Coordinated Science Laboratory
University of Illinois at Urbana-Champaign

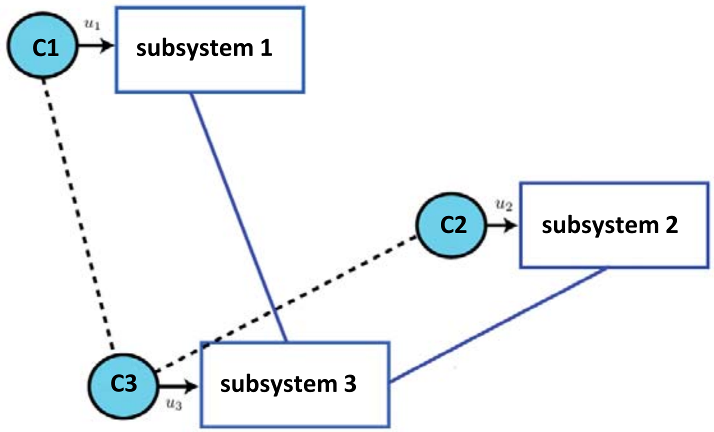
Outline

- **Brief Literature Overview**
- **Decoupled Design for Distributed Control of Uncertain Networked Control Systems**
- **Cooperative Missions of UAVs**

Objectives of Distributed Multi-Agent Systems

Distributed Stabilization

- Weak physical coupling
  - Decentralized control – Siljak 1991
- Strong physical coupling
  - Distributed control – Bamieh 2002 TAC, D'Andrea 2003 TAC, Dunbar 2007 TAC, Rice 2009 TAC



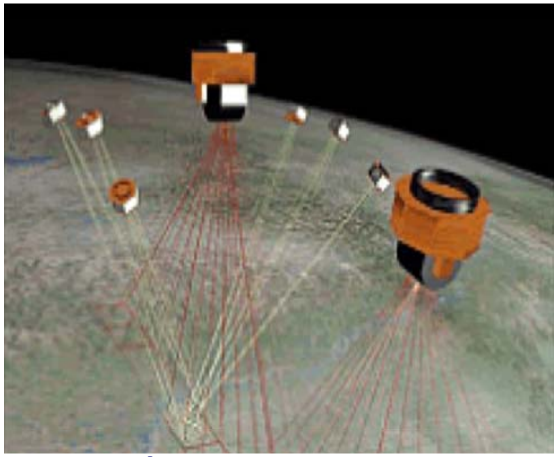
Cooperative Control

- Consensus – Olfati-Saber 2005CDC, Ren 2007CS
- Flocking – Olfati-Saber 2006TAC, Tanner 2003CDC, Blondel 2005CDC
- Formation – Tabuada 2001ACC, Tanner 2002IFAC, Egerstedt 2001TRA
- Maximal coverage – Bullo 2005CC, Cortes 2004TRA, Martinez 2007CSM
- Optimization – Rabbat 2004 IPSN, Palomar 2007TAC, Lee 2004CL

> **Perfect Communication**
>
> **Perfect Model**
>
> - No Physical Uncertainty

Motion Induced Communication

- Flocking:
  - Increase the chance of detecting enemies or targets



- Self-assembled network (Olfati-Saber 2006)
  - No assumptions on network topology (but needs a common objective for the group), more realistic for low cost agents with limited communication range
  - Ideal double integrator dynamics
  - Uncertain dynamics (Li ACC 2011)

- Nonholonomic agents (more realistic and challenging)
  - Tanner 2004 : inter-agent velocity alignment, assumptions on network topology

Main Contributions

- Multiple nonholonomic agents
  - ✓ Uncertain dynamics and disturbances

- Common goal: target tracking

- Self-assembled flocking and collision avoidance

- Cascaded control structure
  - ✓ Coordination and uncertainties decoupled and addressed independently
  - ✓ Guaranteed performance bounds

Problem Formulation

- *N* Mobile (heterogeneous) agents

$$\dot{q}^i(t) = \begin{bmatrix} \dot{x}^i(t) \\ \dot{y}^i(t) \end{bmatrix} = v^i(t) \begin{bmatrix} \cos(\theta^i(t)) \\ \sin(\theta^i(t)) \end{bmatrix}$$

$$\dot{\theta}^i(t) = \omega^i(t)$$

$$v^i(s) = G_v^i(s)(u_v^i(s) + z_v^i(s))$$

$$\omega^i(s) = G_\omega^i(s)(u_\omega^i(s) + z_\omega^i(s))$$

- Target: constant velocity, q_t, θ_t, v_t are known to agents

- Communication: inter-agent communication range r

- Objective: track the target forming a flock and avoid collision

Preliminaries from graph theory

- General definition

- Graph induced by agents' positions

Cascaded Control Structure :: Overview

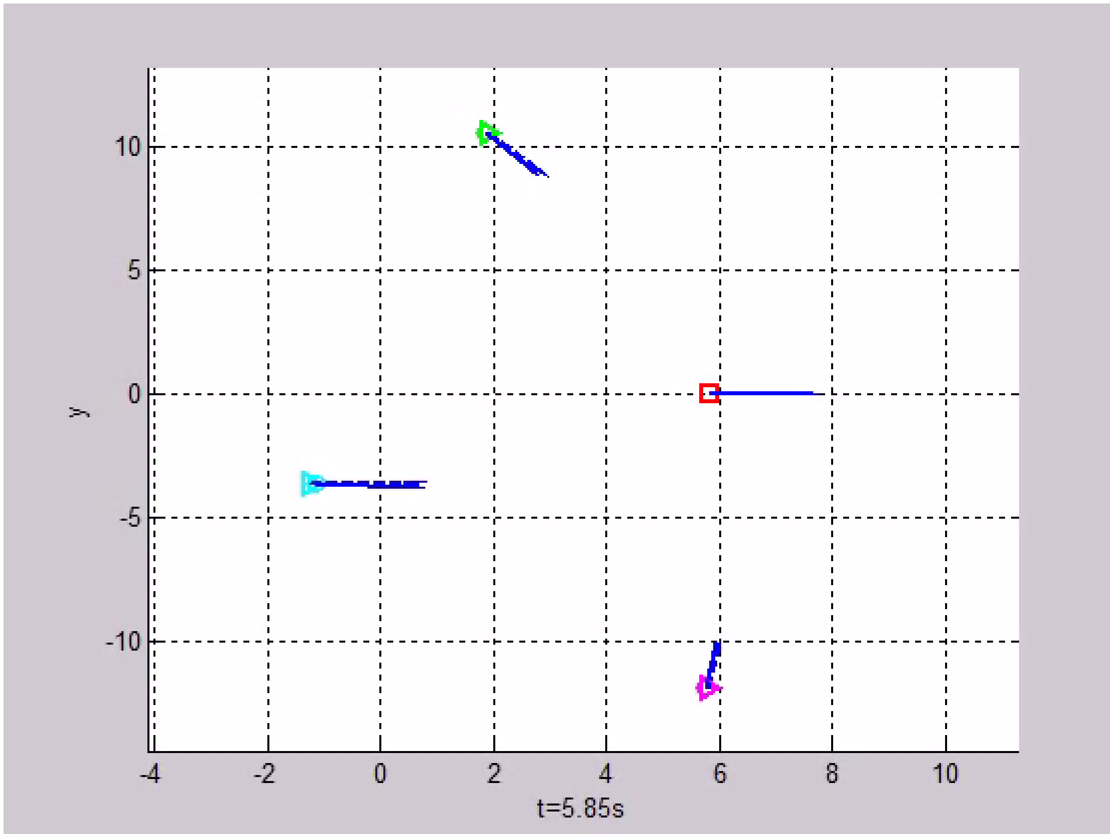
"Simulator" (coordination) + Local Tracking Law (uncertainties)

- Simulator: virtual ideal kinematic model
  - Running in each agent's computer
  - Driven by a cooperative guidance law that achieves flocking and collision avoidance
  - Each agent exchanges information of its simulator with neighbors; this information is uncertainty-free, and is used in the guidance law
  - Position and velocity of the simulator serve as reference inputs for the real agent to track

- Local tracking controller : track the simulator
  - Outer-loop : guidance law for the kinematic model
  - Inner-loop: adaptive controller for uncertain dynamics

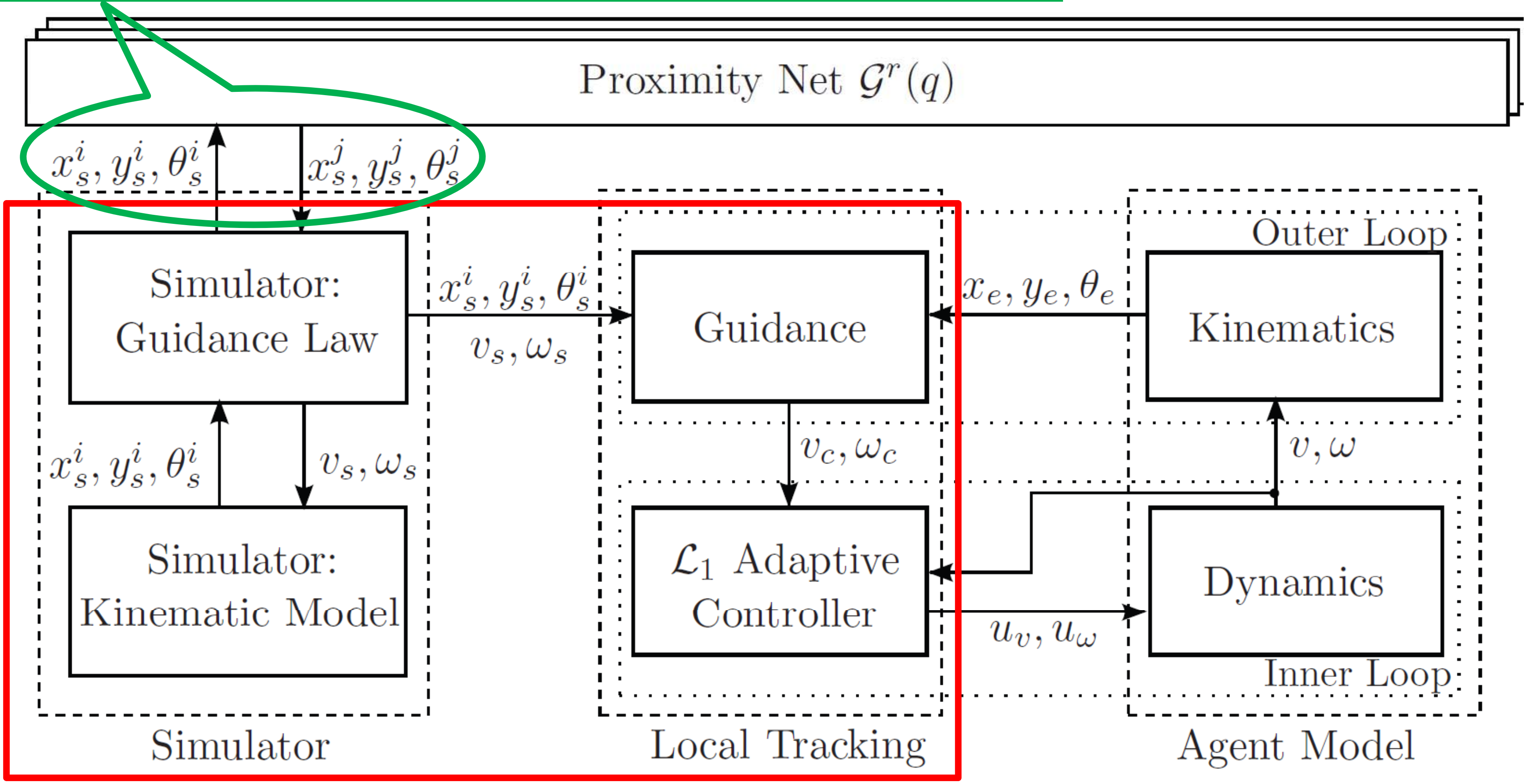
Cascaded Control Structure :: Overview

- A demo of the simulator-follower structure



Cascaded Control Structure :: Overview

Exchange (uncertainty-free) simulator states



Controller

Simulator : Flocking Algorithm for Ideal Kinematic Model

- Ideal kinematic model

$$\dot{q}_{\rm s}^i = \begin{bmatrix} \dot{x}_{\rm s}^i \\ \dot{y}_{\rm s}^i \end{bmatrix} = v_{\rm s}^i \begin{bmatrix} \cos\theta_{\rm s}^i \\ \sin\theta_{\rm s}^i \end{bmatrix}$$

$$\dot{\theta}_{\rm s}^i = \omega_{\rm s}^i$$

$$x_{\rm s}^i(0) = x^i(0)\,, y_{\rm s}^i(0) = y^i(0)\,, \theta_{\rm s}^i(0) = \theta^i(0)$$

- Flocking algorithm

$$v_{\rm s}^i = v_{\rm t} - k_v \nabla_{q_{\rm s}^i} V^i(q_{\rm s}) \cdot [\cos\theta_{\rm s}^i \quad \sin\theta_{\rm s}^i]^\top$$

$$\omega_{\rm s}^i = -k_\omega(\theta_{\rm s}^i - \theta_{\rm d}^i)$$

Smooth transition from aggregation to heading alignment

$$\boxed{\theta_{\rm d}^i} = \text{angle}(-\nabla_{q_{\rm s}^i} V^i(q_{\rm s}) + \vec{v}_{\rm t})$$

- Collective potential function

$$V^i(q_{\rm s}) = \sum_{j \in \mathcal{N}_i^r(q)} V_{\rm a}(\|q_{\rm s}^i - q_{\rm s}^j\|) + V_{\rm t}(\|q_{\rm s}^i - q_{\rm t}\|)$$

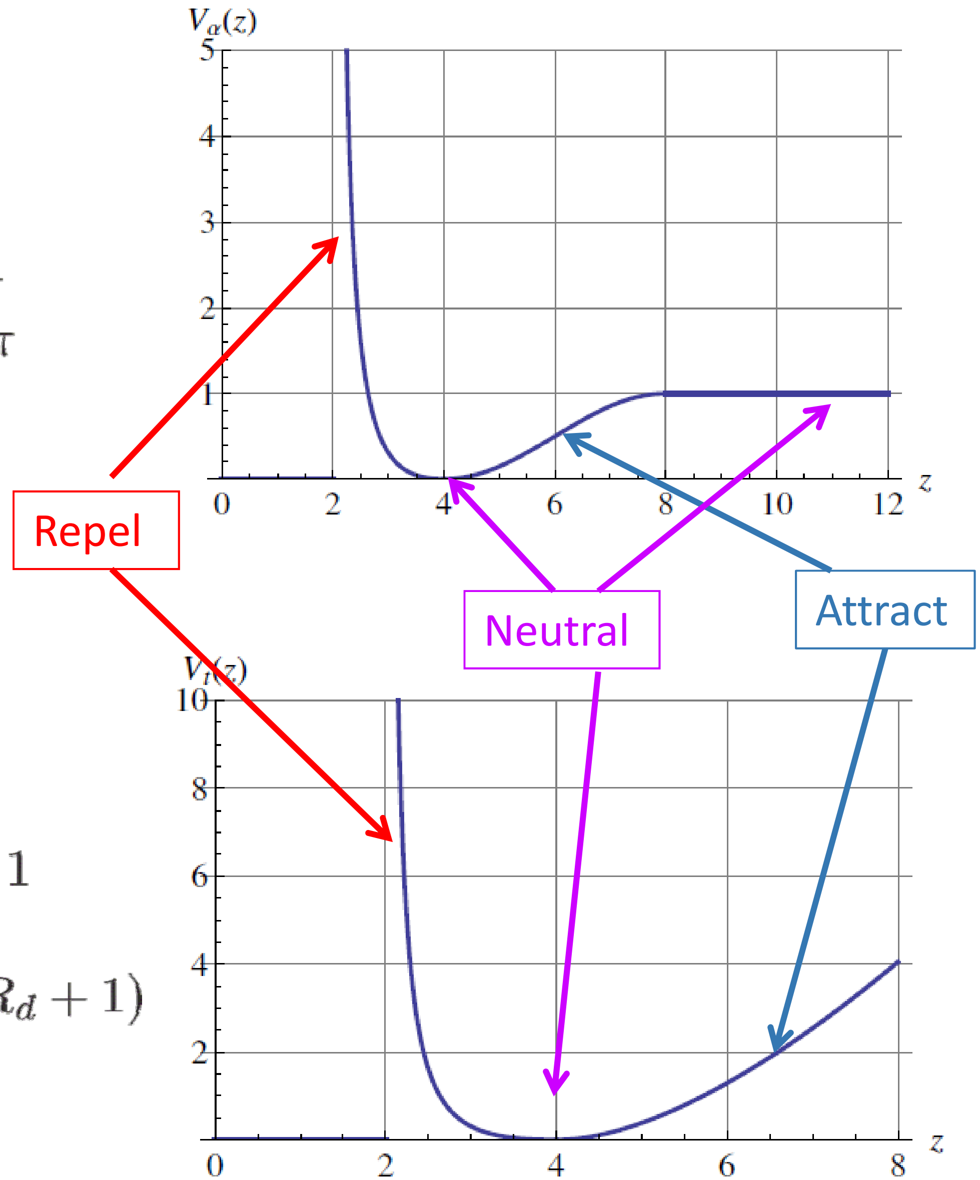
Potential Functions

- Inter-agents

$$V_{\mathrm{a}}(z) = \begin{cases} \infty \\ \log \frac{z - r_c}{r_d - r_c} + \frac{r_d - r_c}{z - r_c} - 1 \\ h + h \sin \frac{r - (r_c + r_d)/2}{r_d - r_c} \pi \\ h \end{cases}$$

- Target-agent

$$V_{\mathrm{t}}(z) = \begin{cases} \infty \\ \log \frac{z - R_c}{R_d - R_c} + \frac{R_d - R_c}{z - R_c} - 1 \\ -(z - R_d) + \\ (z - R_d + 1) \log(z - R_d + 1) \end{cases}$$



Track the Simulator : Outer-loop

- Objective : the real agent tracks the simulator, i.e., $|p_e| \rightarrow 0$

- Tracking error

$$p_e = \begin{bmatrix} x_e \\ y_e \\ \theta_e \end{bmatrix} = \underbrace{\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{T} \begin{bmatrix} x_s - x \\ y_s - y \\ \theta_s - \theta \end{bmatrix}, \quad \det(T) \neq 0.$$

$$\dot{p}_e = \begin{bmatrix} \dot{x}_e \\ \dot{y}_e \\ \dot{\theta}_e \end{bmatrix} = \begin{bmatrix} \omega y_e - v + v_s \cos\theta_e \\ -\omega x_e + v_s \sin\theta_e \\ \omega_s - \omega \end{bmatrix}$$

- Velocity commands (Kanayama et al, 1991)

$$u_c = \begin{bmatrix} v_c \\ \omega_c \end{bmatrix} = \begin{bmatrix} k_x x_e + v_s \cos\theta_e \\ \omega_s + k_y y_e + k_\theta \sin\theta_e \end{bmatrix}$$

Track the Simulator : Outer-loop

- For perfect velocity tracking: $v = v_c, \omega = \omega_c$

$$\dot{p}_e = f(p_e\,, u_s) \triangleq \begin{bmatrix} (\omega_s + k_y y_e + k_\theta \sin\theta_e) y_e - k_x x_e \\ -(\omega_s + k_y y_e + k_\theta \sin\theta_e) x_e + v_s \sin\theta_e \\ -k_y y_e - k_\theta \sin\theta_e \end{bmatrix}$$

  - v_s, ω_s constant: asymptotic stability

- Key result: imperfect velocity tracking:

  - If $|v - v_c| < \gamma_v, |\omega - \omega_c| < \gamma_\omega$, then $|p_e| < \gamma_e$, where

Track the Simulator : Inner-loop

- **Objective:** $v(s) \approx M_v(s) v_c,\ \omega(s) \approx M_\omega(s)\omega_c(s)$

$$v^i(s) = G_v^i(s)(u_v^i(s) + z_v^i(s))$$
$$\omega^i(s) = G_\omega^i(s)(u_\omega^i(s) + z_\omega^i(s))$$

- Solution: L1 Adaptive Controller

- Main features of L1 Adaptive Controller

  - **Guaranteed transient response for system's both input and output:**

Track the Simulator : Inner-loop

- L1 adaptive controller for v, $M_v(s) = \frac{m_v}{s+m_v}$
  - Output predictor: $\dot{\hat{v}}(t) = -m_v \hat{v}(t) + m_v(u_v(t) + \hat{\sigma}_v(t))$
  - Adaptive law: $\dot{\hat{\sigma}}_v(t) = \Gamma_v \mathbf{Proj}(\hat{\sigma}_v(t), -\tilde{v}(t))$
  - Control law: $u_v(s) = C_v(s)(v_c(s) - \hat{\sigma}_v(s))$
  - L1 stability condition: $H_v(s) \triangleq \frac{G_v(s)M_v(s)}{C_v(s)G_v(s)+(1-C_v(s))M_v(s)}$ stable

    $\|H_v(s)(1 - C_v(s))\|_{\mathcal{L}_1} L_v < 1$

- Key result: If v_c is bounded and $\dot{v}_c \leq \gamma_{\dot{v}_c}$, then

$$\|v - v_c\|_{\mathcal{L}_\infty} \leq \frac{\gamma_v^{\text{ref}}}{\sqrt{\Gamma_v}} + \gamma_v^{\text{des}} + \frac{1}{m_v}\gamma_{\dot{v}_c}$$

Short Summary

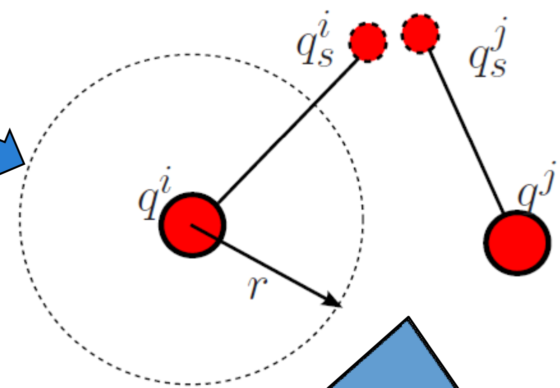
- $|v - v_c|, |\omega - \omega_c|$ small $\Rightarrow |p_e|$ bounded $\forall t$
- L1 adaptive controller: $|v - v_c|_{L_\infty}, |\omega - \omega_c|_{L_\infty}$ small

Agent can track the simulator with transient performance guarantees!

Coupling between dynamics and topology

- Coupling exists because:
  - **Communication is induced by motion, no artificial assumptions**
  - **Input to the simulator is based on $\mathcal{G}^r(q)$, not $\mathcal{G}^r(q_s)$**

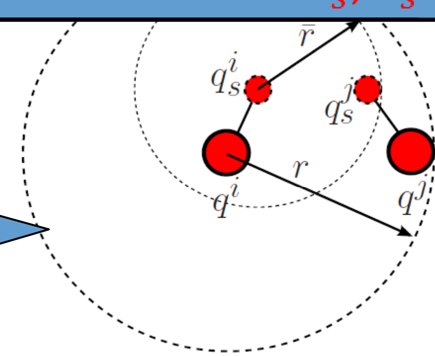
➢ Without transient guarantees, it is possible that simulators "move" to the same position



➢ **With transient guarantees, we can select**

$$(i,j) \in \mathcal{E}^{\bar{r}}(q_s) \Rightarrow (i,j) \in \mathcal{E}^r(q)$$

Real agents can't talk, simulators "collide", and generate infinite v_s, ω_s



- The guaranteed transie the controller is the k coupling issue.

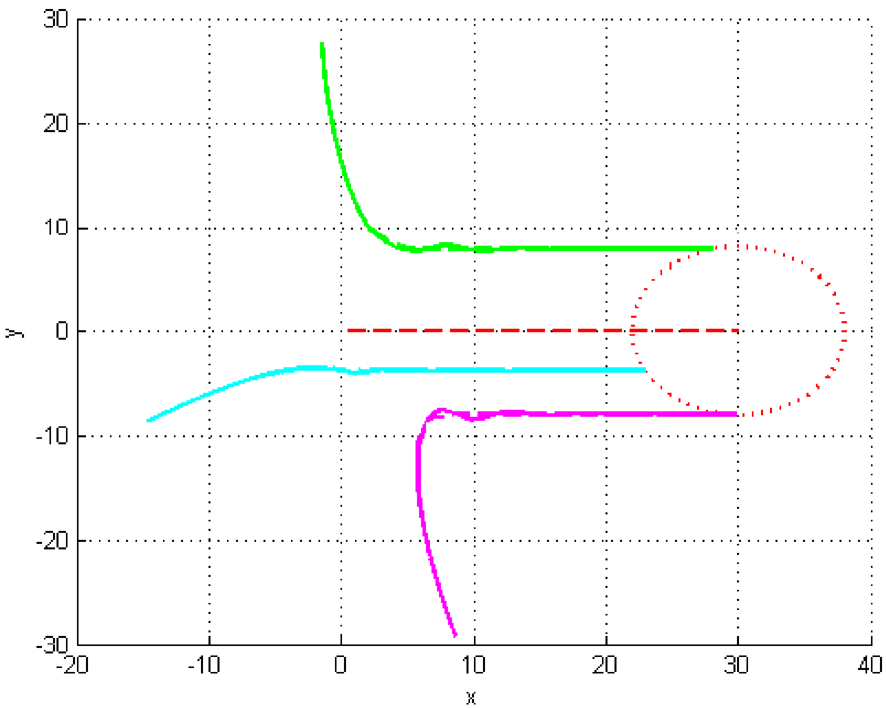
Real agents are always close to simulator, and well behaving

Simulation: 3 agents, without L1 adaptive controller

- Algorithm designed for ideal dynamics

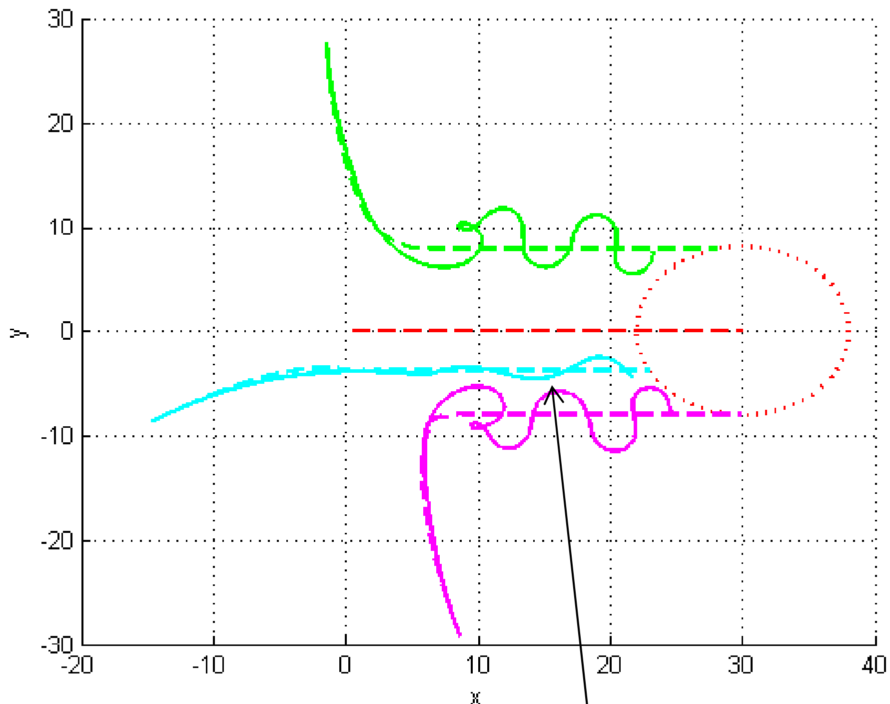
Response for ideal dynamics

$$G_v^i(s) = G_\omega^i(s) = \frac{1}{s}, z_v^i = z_\omega^i = 0$$



Response for slow dynamics

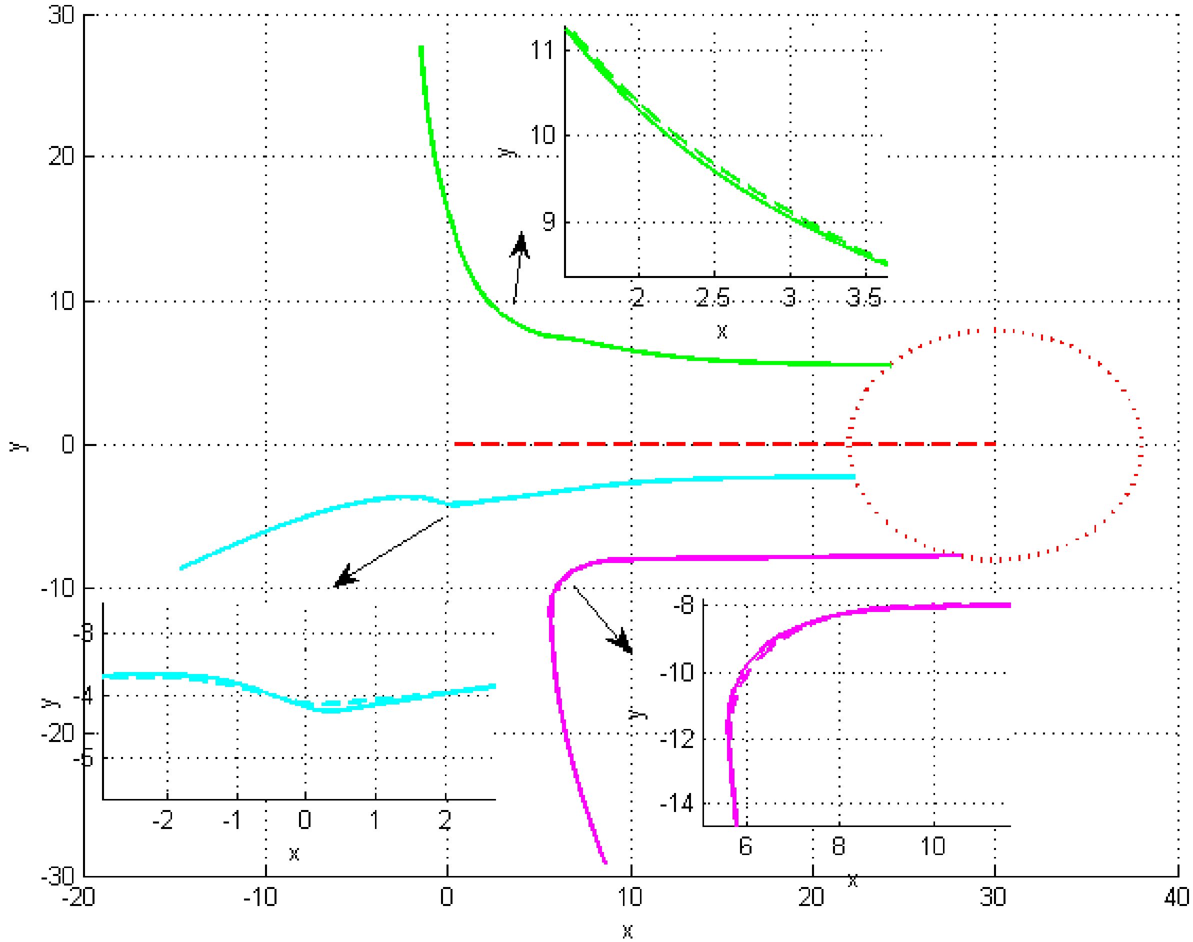
$$G_v^i(s) = G_\omega^i(s) = \frac{0.2}{s+0.2}, z_v^i = z_\omega^i = 0$$



Almost collide

Simulation: 3 agents, with L1 adaptive controller

- Trajectories



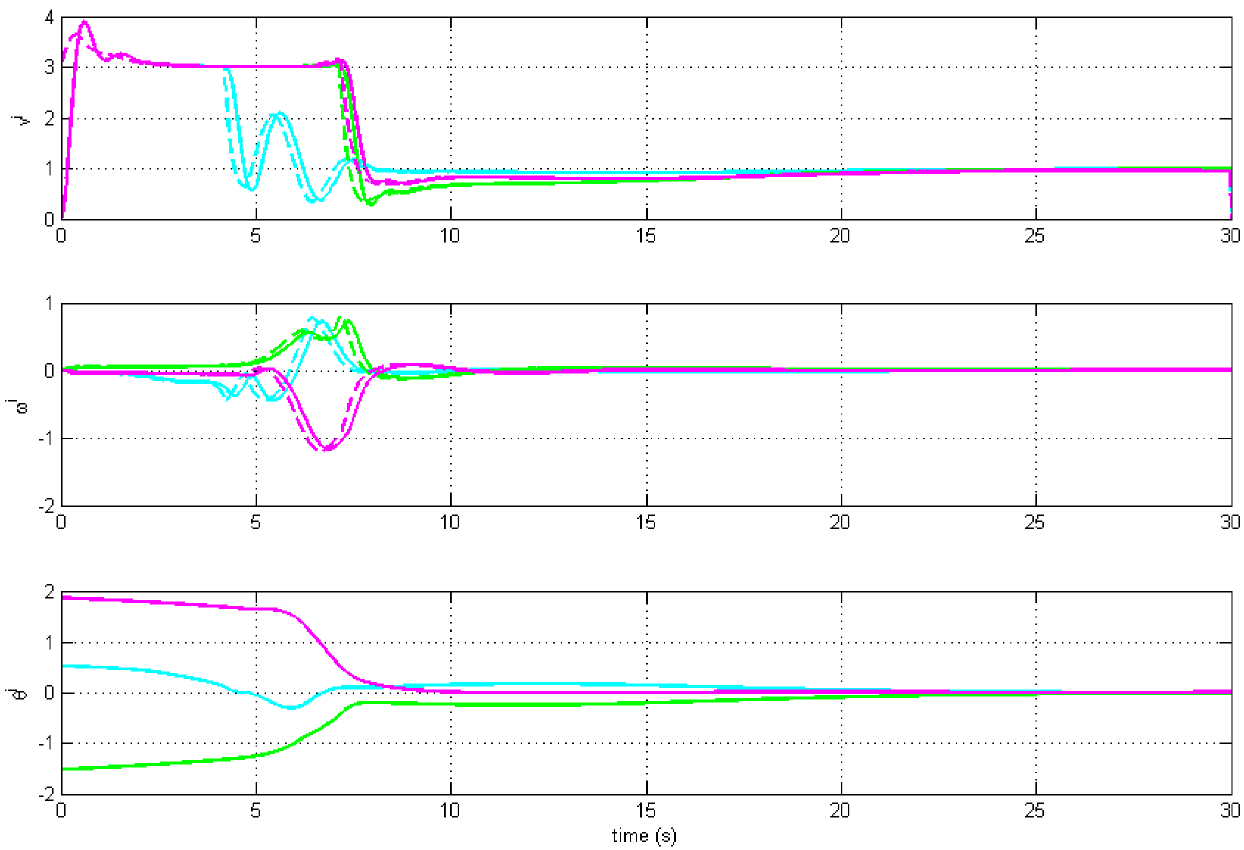
Simulation: 3 agents, with L1 adaptive controller

- Velocity tracking

v^i, v_c^i

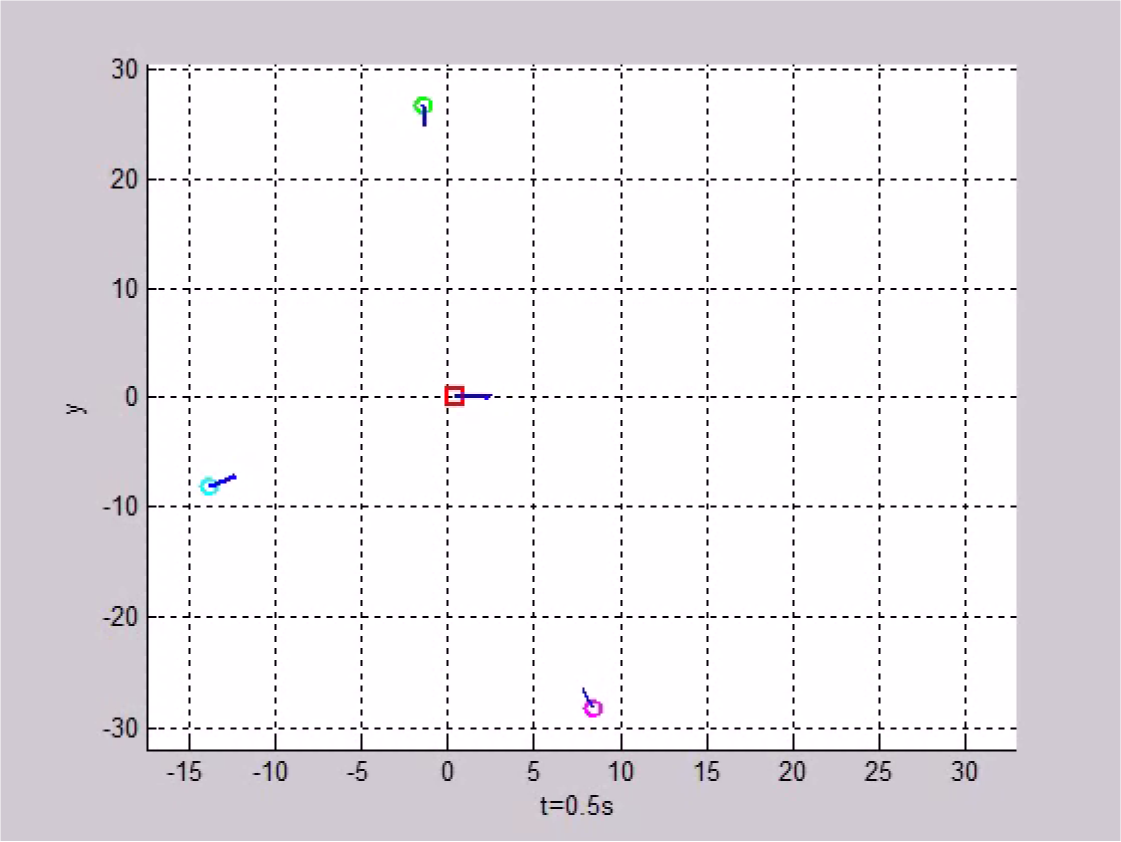
ω^i, ω_c^i

θ^i



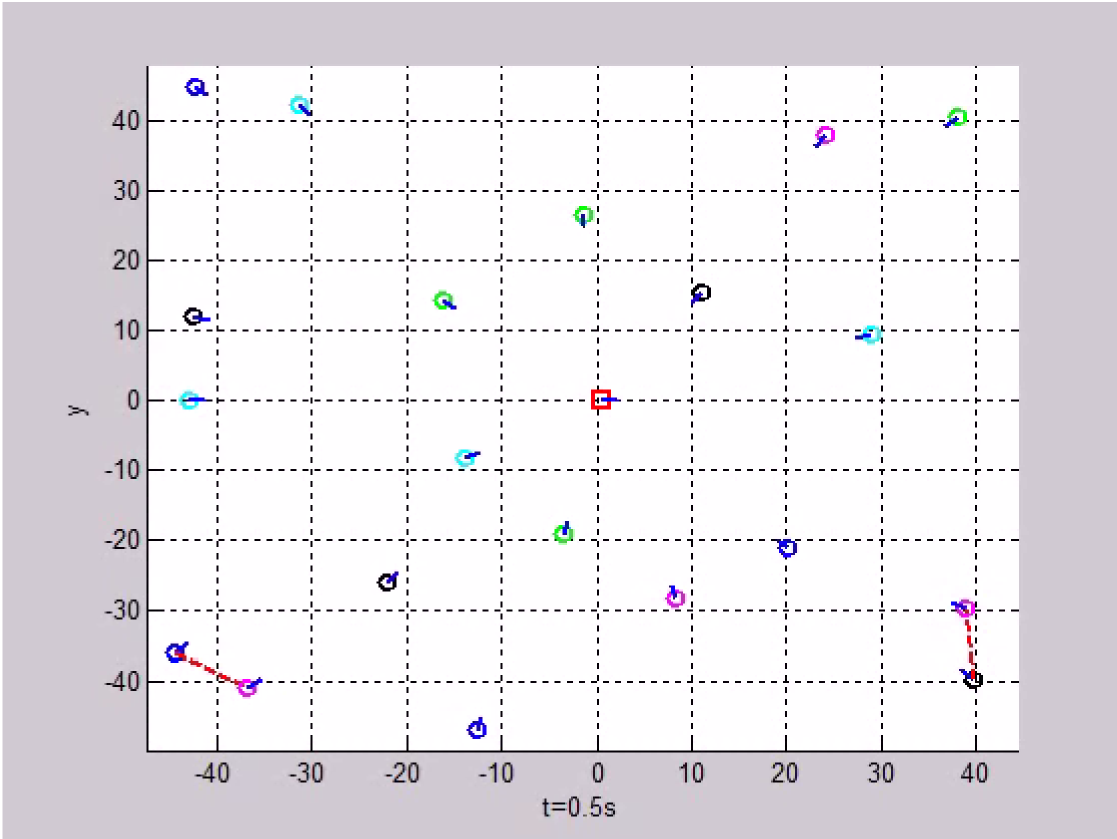
Simulation: 3 agents, with L1 adaptive controller

- Animation



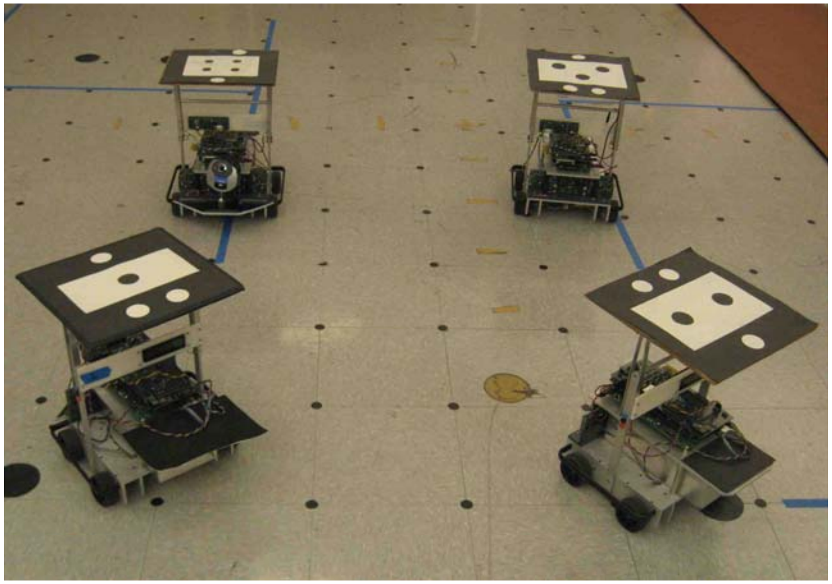
Simulation: 20 agents, with L1 adaptive controller

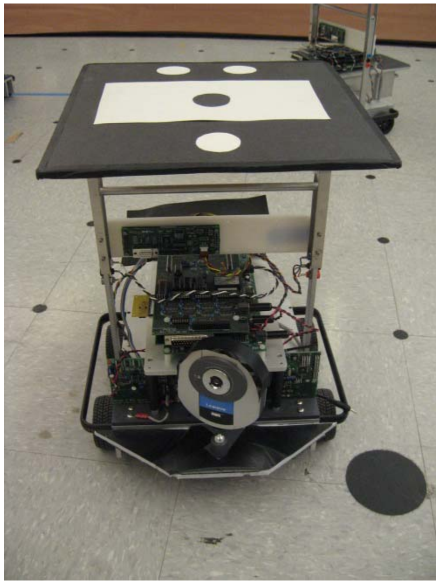
- Animation



Experimental results

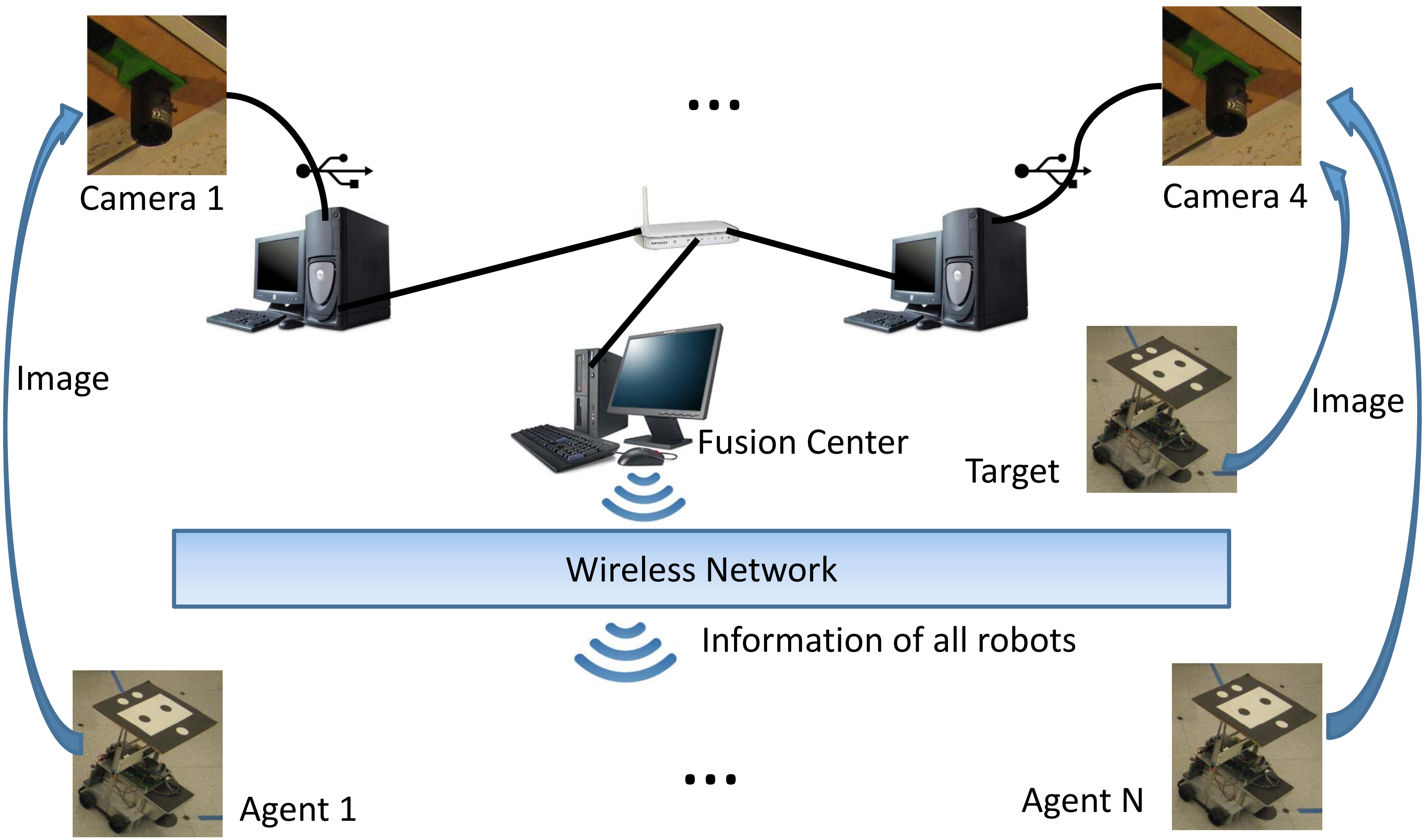
- Ground robots





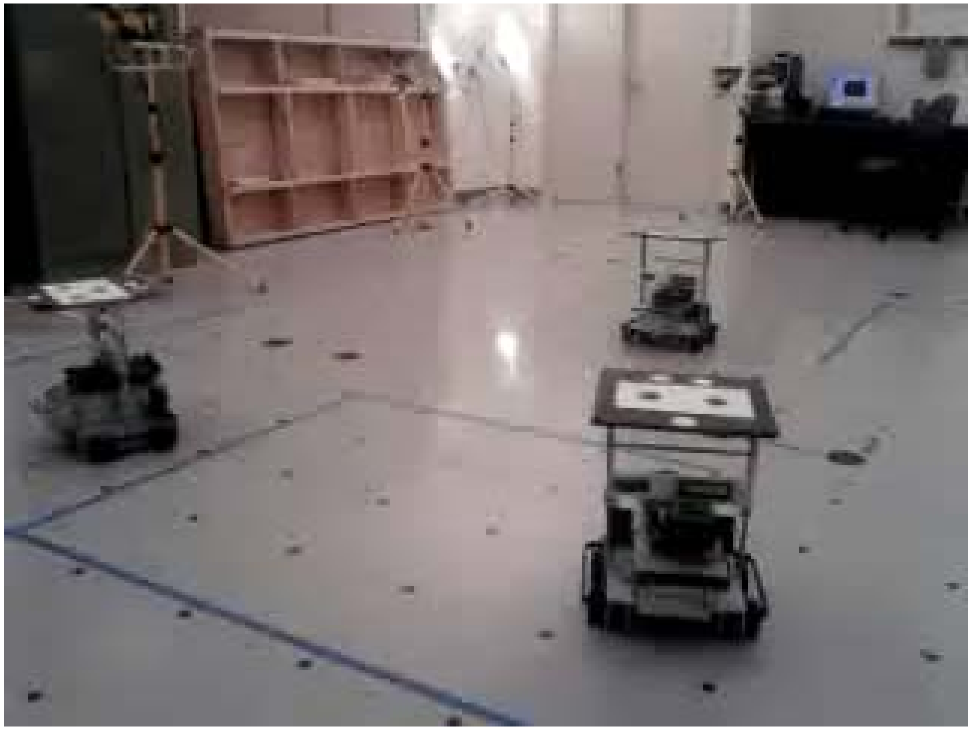
Experimental results

- Experimental configuration

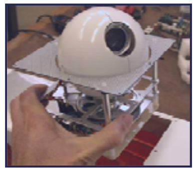


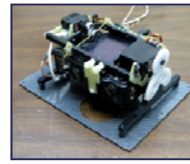
Experimental results

- A test run with 2 followers



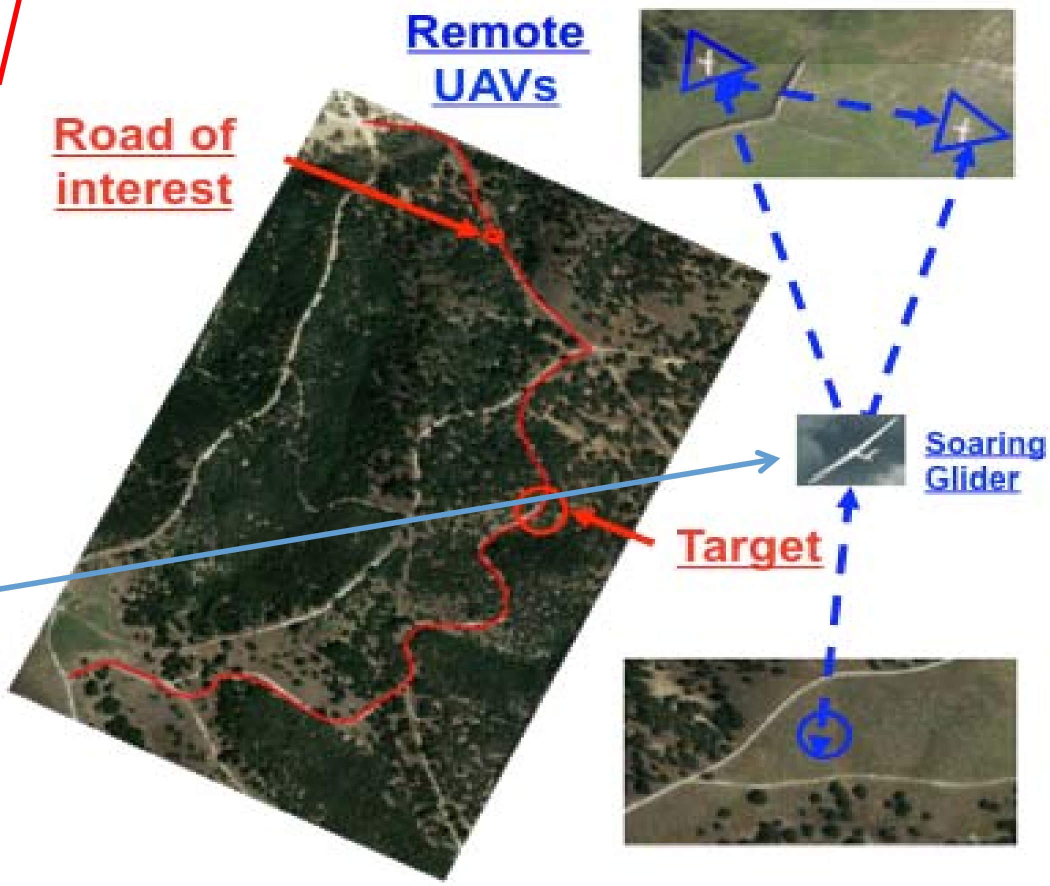
Coordinated Road Search and Target Tracking – The Concept





Single DOF gimbal with *high resolution camera* delivers satellite quality imagery well suited for *target ID*

2DoF P/T gimbal with a *video camera* enables vision based UAV guidance and *target tracking*

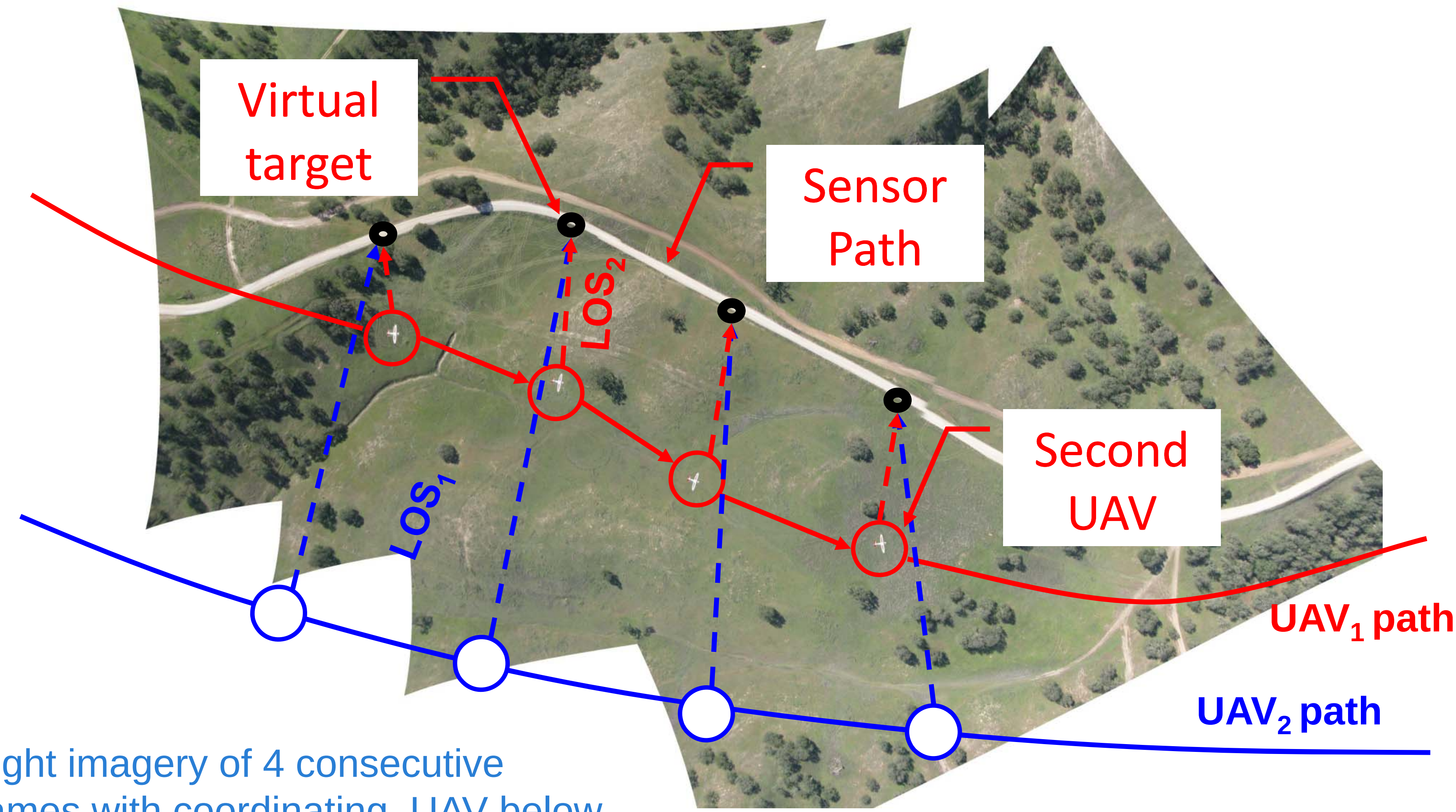


Thermal seeking *soaring gliders is used as a Flying Antenna to extend communication range*

Flight Test: Coordinated Road Capture and Search

Coordination in "Road Search" mode (CPF) is represented by the results of stitching 4 consecutive high resolution frames taken from the UAV above:

- UAVs are looking at the same virtual target on the road
- Successful coordination results in "encapsulating" second UAV in each frame

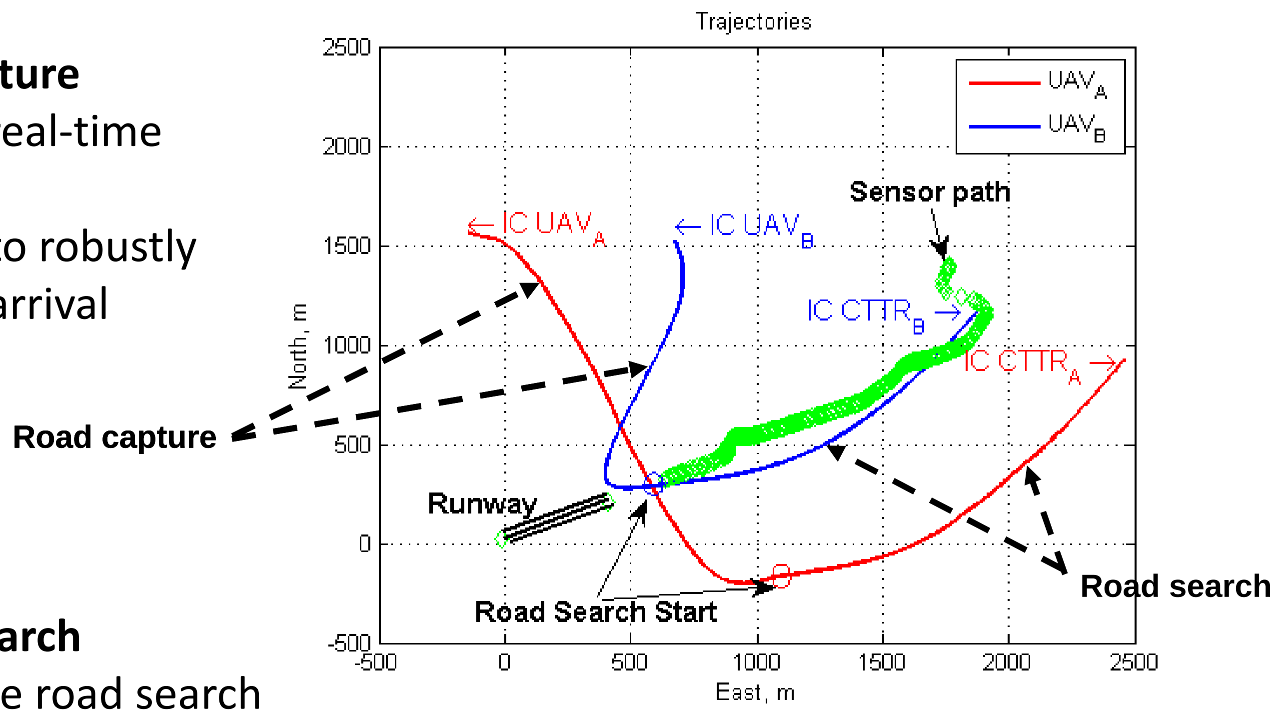


Flight imagery of 4 consecutive frames with coordinating UAV below

Flight Test: Coordinated Road Capture and Search

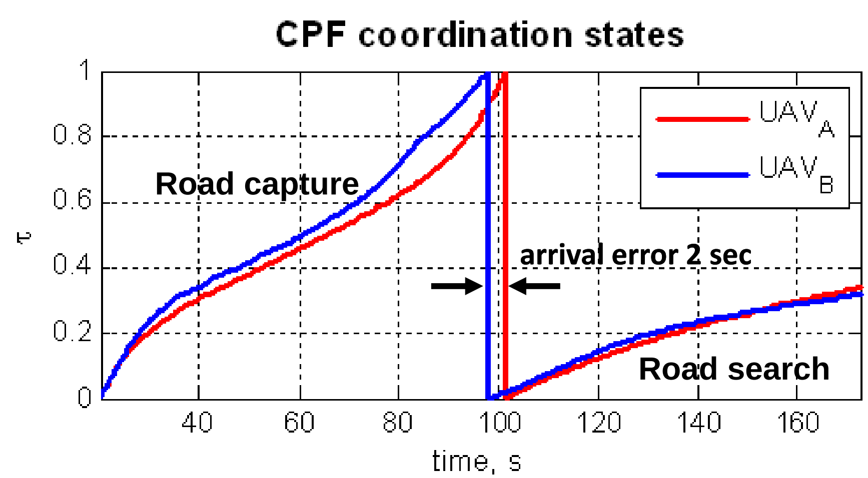
Coordinated Road Capture

- two UAVs generate real-time capture paths
- coordination is used to robustly achieve simultaneous arrival



Coordinated Road Search

- two UAVs follow the road search paths
- coordination is used to guarantee nonzero intersection of the FOVs between two cameras



Flight Test: Coordinated Road Capture and Search

If coordination is successful then both cameras look at the same place

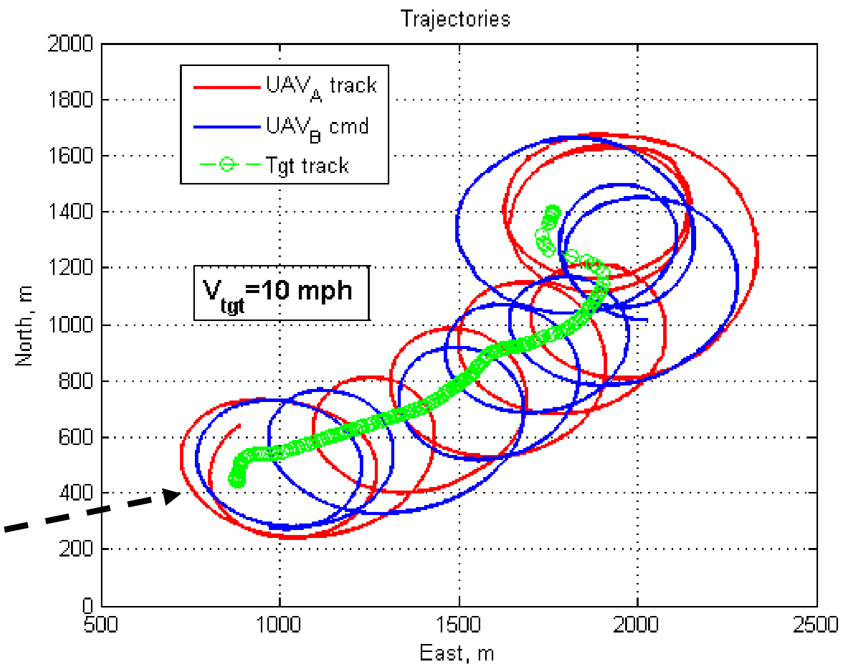
The video at the top is from the UAV with a standard 2DOF gimbaled camera

The video at the bottom was generated by the high rez camera stabilized by a 1DOF gimbal (this was accounted for in the path generation)

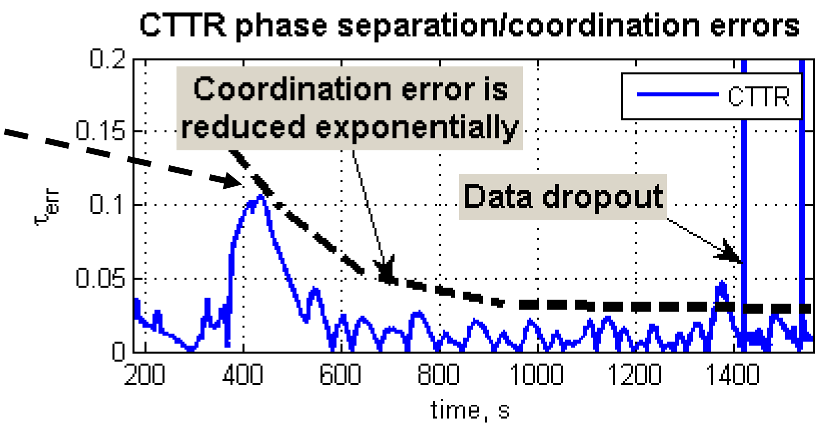
Flight Test: Coordinated Target Tracking

Once the target is designated by the user

• two UAVs switch from road search to coordinated target tracking mode

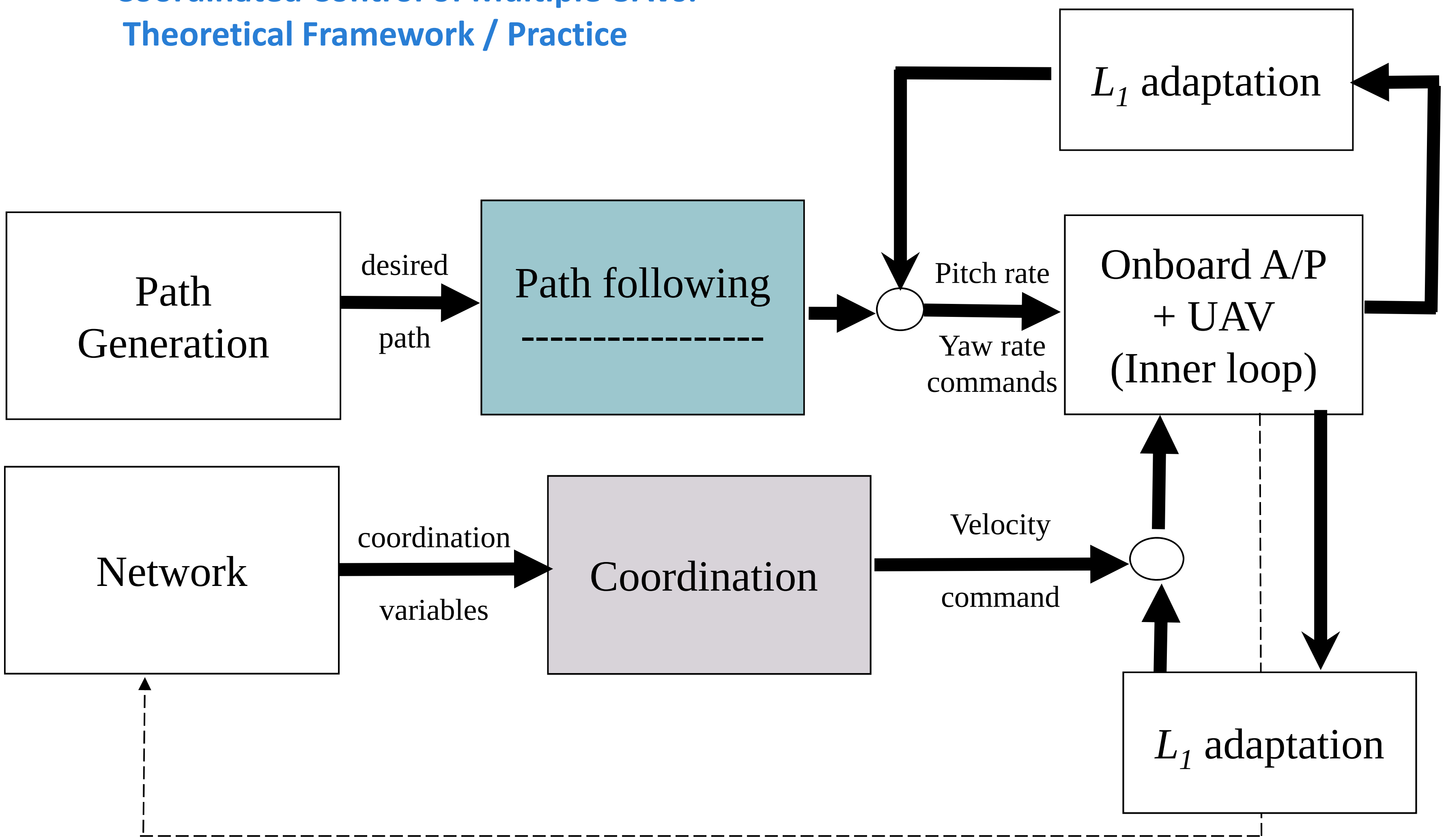


• the pre-assigned phase separation is pi/2, maintained by coordination



Conclusions for Coordinated Road Search

- **Coordinated Control of Multiple UAVs: Theoretical Framework / Practice**



Real-Time Issues

Limited Communication Resource

- Transmission delays, discrete data transmission, packet loss, quantization

Sampled-Data Systems (Single Plant)

- Periodically sampled system --- Chen 1995
- Event-triggered control --- Årzén 1999IFAC, Tabuada 2007TAC
  - Saves Communication Resource, but requires hardware detector
- Self-triggered control --- Velasco 2002RTSS, Wang 2009TAC 2010TAC, Anta 2010TAC
  - Predict the next execution time based on the past information. No need for hardware!

Real-Time Communication in Multi-Agent Systems

- Physically Decoupled
  - Consensus, Dimarogonas 2009CDC, 2011IFAC, Yu2011CDC
  - Maximum Coverage, Nowzari, 2011Automatica 2012ACC
- Physically Coupled
  - Stabilization, Wang 2009TAC 2011TAC
  - Network Utility Maximization, Wan 2009IPSN

- **Perfect Model**
  - No Physical Uncertainty

Robust Control and Communication Co-Design

- Input-to-state stability for robustness
  - Periodic Transmission – Nesic 2004TAC
  - Event-triggering broadcast – Wang 2008ACC, Mazo 2009CDC
  - ISS only provides an upper bound on the state trajectory, without quantifying the *transient performance*.
- L_1-adaptation-based event-triggering for performance
  - State Feedback, linear interconnection, matched uncertainty, fixed communication model – (presented in AFOSR in 2010)

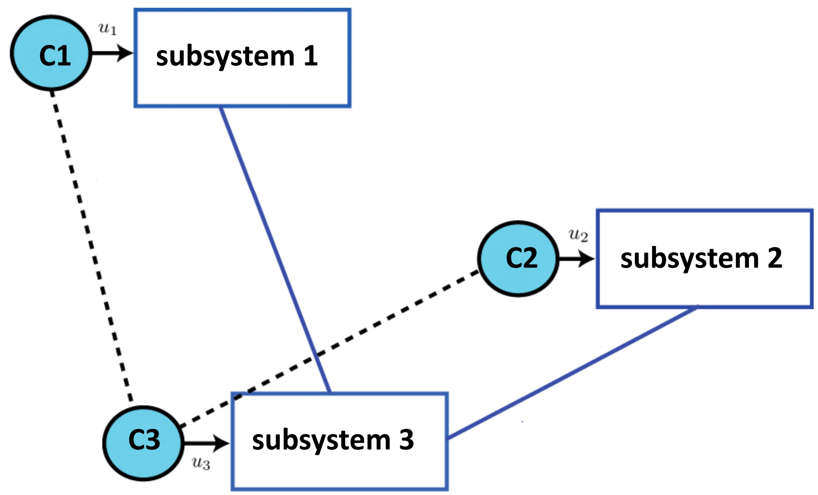
Drawback: Event-triggering was defined to transmit the **real system outputs** over the network, often measured by noisy sensors.

Desired Solution:

*1.Network must be used for communication of information, **NOT affected** by system uncertainties or noise.*

2.The design of control and communication can be decoupled to the maximum extent so that the existing techniques can be easily leveraged.

Problem Formulation



Ideal Model:

$$\begin{aligned} \dot{x}_i^{\mathrm{id}} &= A_i^m x_i^{\mathrm{id}} + B_i u_i^{\mathrm{id}} + f_i\left(t, y_i^{\mathrm{id}}, y_{-i}^{\mathrm{id}}\right) \\ y_i^{\mathrm{id}} &= C_i x_i^{\mathrm{id}}, \quad x_i^{\mathrm{id}}(0) = x_i^0 \end{aligned}$$

Ideal Controller:

$$\begin{aligned} \dot{z}_i^{\mathrm{id}} &= g_i\left(t, z_i^{\mathrm{id}}, y_i^{\mathrm{id}}, y_{-i}^{\mathrm{id}}\right) \\ u_i^{\mathrm{id}} &= h_i\left(t, z_i^{\mathrm{id}}, y_i^{\mathrm{id}}, y_{-i}^{\mathrm{id}}\right) \end{aligned}$$

Real Subsystem:

$$\begin{aligned} \dot{x}_i &= A_i x_i + B_i u_i + \Delta_i(t, y) \\ y_i &= C_i x_i, \quad x_i(0) = x_i^0, \end{aligned}$$

Input is computed based on discrete data over networks

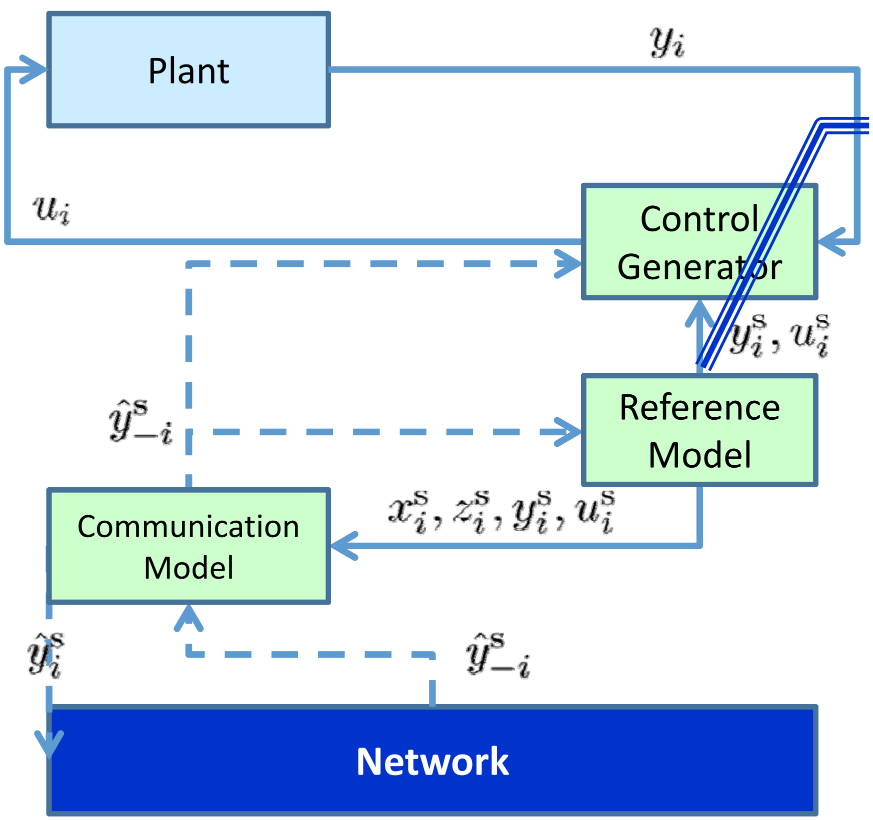
Uncertainties

- Subsystems are physically coupled
  - *Unmatched uncertainties* in the system dynamics and interconnections
  - Uncertainties are Locally Lipschitz
  - Output feedback
  - Nonlinear ideal interconnection
- Limited Communication
  - Discrete data transmission
  - Delay and packet loss may exist

Decoupled Control Architecture

To what level can we decouple the design of control and communication?

Solution:



Reference Model

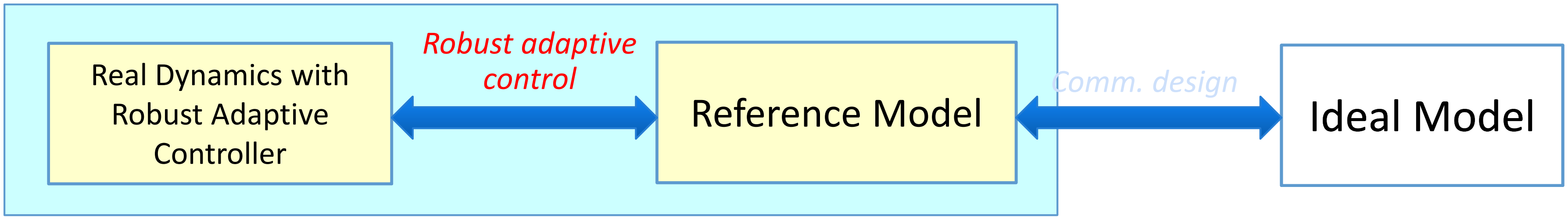
$$
\begin{aligned}
\dot{x}_i^s &= A_i^m x_i^s + B_i u_i^s + f_i\left(t, y_i^s, \hat{y}_{-i}^s\right) \\
y_i^s &= C_i x_i^s \\
x_i^s(0) &= x_i^0 \\
\dot{z}_i^s &= g_i\left(t, z_i^s, y_i^s, \hat{y}_{-i}^s\right) \\
u_i^s &= h_i\left(t, z_i^s, y_i^s, \hat{y}_{-i}^s\right)
\end{aligned}
$$

Network Data

- Doesn't contain physical uncertainties.
- The information is "clean".

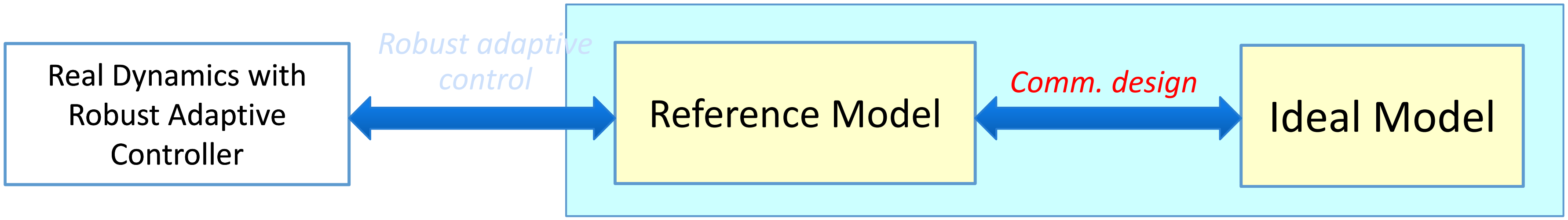
local data based on the data from the reference model.

Control Design



- Regard the real subsystem as a single system, with the signals from its reference model as the reference signals;
- Robust adaptive control ensures the closeness between signals inside the real dynamics and the reference model inside each individual agent.

Communication Design



- Regard the real subsystem as a single system, with the signals from its reference model as the reference signals;
- Robust adaptive control ensures the closeness between signals inside the real dynamics and the reference model inside each individual agent.

- No physical uncertainties are involved in the communication model design;
- If the signals over the network are bounded, the closeness between the real dynamics and the reference model will ***NOT*** be affected by
  - The type of communication models
  - The network effects, such as delays, packet loss, etc.

Design Procedure

Given a multi-agent system

1. Model the system dynamics and obtain the ideal model
2. Design the ideal control algorithm based on the ideal model
3. Design the communication model based on the ideal model so that
    - The resulting closed-loop system (reference model) fulfills the same objective that the ideal model does
    - The input and the output of the resulting reference system are bounded.
4. With the bounds obtained in Step 3, design the distributed local robust adaptive controller

The only coupling between the control and the communication design is that the bounds on the input and the output of the reference model will be used in selecting the parameters of the control generator.

The Difference between Reference and Ideal Models

The reference model:

$$\begin{aligned} \dot{x}_i^s &= A_i^m x_i^s + B_i u_i^s + f_i\left(t, y_i^s, \hat{y}_{-i}^s\right) \\ y_i^s &= C_i x_i^s \\ \dot{z}_i^s &= g_i\left(t, z_i^s, y_i^s, \hat{y}_{-i}^s\right) \\ u_i^s &= h_i\left(t, z_i^s, y_i^s, \hat{y}_{-i}^s\right). \end{aligned}$$

The combined closed-loop reference model:

$$\dot{w}^s = \Psi\left(t, w^s(t), \hat{y}^s(t)\right)$$

The closed-loop ideal model:

$$\dot{w}^{id} = \Psi\left(t, w^{id}(t), y^{id}(t)\right)$$

The closed-loop reference model can be written as:

$$\dot{w}^s = \underbrace{\Psi\left(t, w^s(t), y^s(t)\right)}_{\text{Ideal Model}} + \underbrace{\Psi\left(t, w^s(t), \hat{y}^s(t)\right) - \Psi\left(t, w^s(t), y^s(t)\right)}_{\text{Pertubation}}$$

The key challenge in the communication model design is to ensure that the difference between $\hat{y}^s$ and y^s is not TOO large.

Performance Bound

Assume that

- $\Psi(t, w^{s}, y^{s})$ and its first partial derivatives are continuous, bounded, and Lipschitz in w^{s}, uniformly in t, for all $t \geq 0$ and w^{id} in a compact set;
- $\Psi(t, w^{s}, \hat{y}^{s})$ is piecewise continuous in t, locally Lipschitz in y^{s}, $\hat{y}^{s}$;
- there exist continuous, positive definite class $\mathcal{K}$ functions α_1, α_2, α_3 and a continuously differentiable function V such that

$$\alpha_1(\|w^{\mathrm{id}}\|) \leq V(t, w^{\mathrm{id}}) \leq \alpha_2(\|w^{\mathrm{id}}\|)$$
$$\frac{\partial V}{\partial t} + \frac{\partial V}{\partial w^{\mathrm{id}}} \Psi\left(t, w^{\mathrm{id}}\right) \leq -\alpha_3(\|w^{\mathrm{id}}\|);$$

- $w^{\mathrm{id}} = 0$ is an exponentially stable equilibrium point of the ideal model.

Then there exist $\beta, \eta \geq 0$, such that if

$$\boxed{\sum_{i \in \mathcal{N}} \|y^{s} - \hat{y}^{s}\|_{\mathcal{L}_\infty} \leq \eta,}$$

Communication Constraints

we have

$$\|w^{\mathrm{id}} - w^{s}\|_{\mathcal{L}_\infty} \leq \beta\eta.$$

Enforcing Communication Constraint

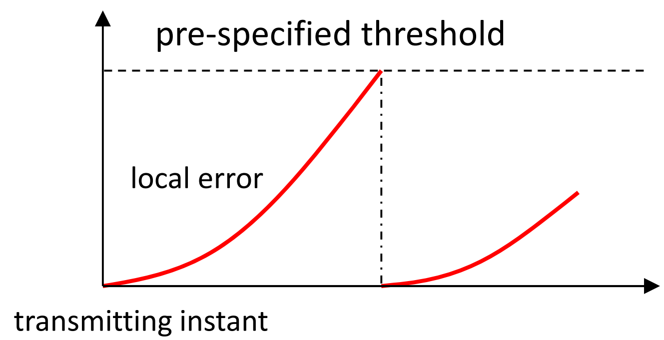
Assume that

Example (to enforce the communication constraint):

Event Triggered Data Transmission:

Transmit when the local measurement error $\|y_i^s(t) - \hat{y}_i^s(t)\|$ exceeds a pre-specified positive constant.

✓The sampled output of the reference model is transmitted. The real output is not involved.



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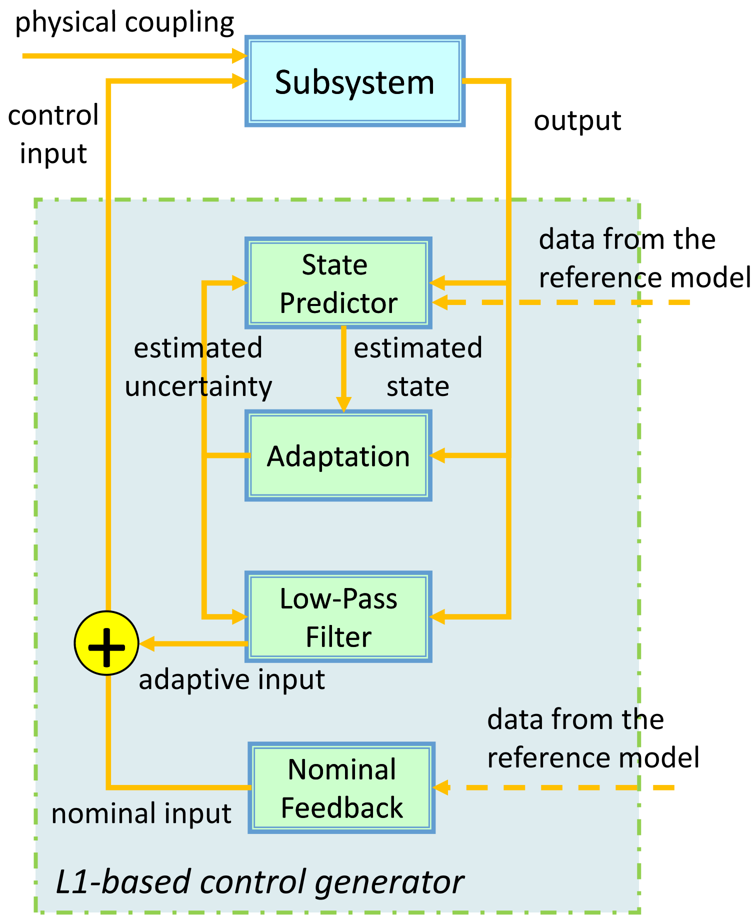
Communication Constraints

we have

$$\|w^{\mathrm{id}} - w^s\|_{\mathcal{L}_\infty} \leq \beta\eta.$$

Local Networked L1 Adaptive Control Generator



- State predictor

$$\dot{\hat{x}}_i = A_i^m \hat{x}_i + B_i u_i + \hat{\eta}_i + f_i\left(t, y_i^s, \hat{y}_{-i}^s\right)$$
$$\hat{x}_i(0) = x_i^0,$$

- Adaptive law

$$\hat{\eta}_i(t) = \Gamma_i(C_i \hat{x}_i(t) - y_i(t))$$

Γ_i: matrix adaptive gain

- Adaptive input

$$v_i^c(s) = -F_i(s)\bar{H}_i(s)^{-1}\hat{H}_i(s)\hat{\eta}_i(s)$$

with $\hat{H}_i(s) = C_i(sI - A_i^m)^{-1}$, $\bar{H}_i(s) = \hat{H}_i(s)B_i$.

- Nominal input

$$v_i^n(t) = u_i^s(t)$$

This L1 adaptive control architecture uses data from the reference model that carries the information from the network.

Main Results

Assume that the input and the output of the ith reference model are bounded by $\rho_i^{u^s}$ and $\rho_i^{y^s}$, respectively. If $\exists \rho_y$ such that

$$\begin{aligned}\rho_y \quad > \quad & \phi_i(\|x_i^0\|) + \psi_i(\rho^{u_i^s}) + \gamma_i\left(\max_{i\in\mathcal{N}_i} \rho_i^{y^s}\right) \\ & + \|\left(I - F_i(s)\right)\hat{H}_i(s)\|_{\mathcal{L}_1}\left(a_i\rho_y + b_i\right) + \frac{1}{\alpha_i(\sigma_{\min}(A_i^m + \Gamma_i C_i))}\end{aligned}$$

for any i, then $\|y(t)\|$ and $\|u(t)\|$ are uniformly bounded and

$$\|y - y^s\|_{\mathcal{L}_\infty} \leq \rho_{\bar{\Delta}_i}\left(\|(I - F_i(s))\hat{H}_i(s)\|_{\mathcal{L}_1} + \frac{1}{\beta(\sigma_{\min}(A_i^m + \Gamma_i C_i))}\right),$$

where ϕ_i, ψ_i, γ_i, α_i, β are class $\mathcal{K}_\infty$ functions and $\sigma_{\min}(\cdot)$ denotes the minimal singular value of a matrix.

- We can always find ρ_y such that the stability condition holds.
- The performance bound can be rendered arbitrarily small by increasing the bandwidth of the low-pass filter.
- Large bandwidth leads to small stability margin, which suggests a tradeoff between performance and robustness.

Real-Time L_1 Adaptive Control Generator

When implementing the controller in digital processors, we seek real-time L1 adaptive controller and consider the problem of computational resource management.

Assume that the $\mathcal{L}_1$ stability condition holds. For each agent, if the time intervals between two consecutive receptions of the data from the network are lower bounded by a positive constant, then there exists a continuous function α_i satisfying $\alpha_i(0,0,0,0,0) = 0$ and a positive constant γ_i such that if

$$\alpha_i \left(\begin{array}{l} \text{Comp. Period; Sensing Period; Actuation Period;} \\ \text{Sensing Delay; Actuation Delay} \end{array} \right) < \gamma_i$$

Sensing Period > Sensing Delay

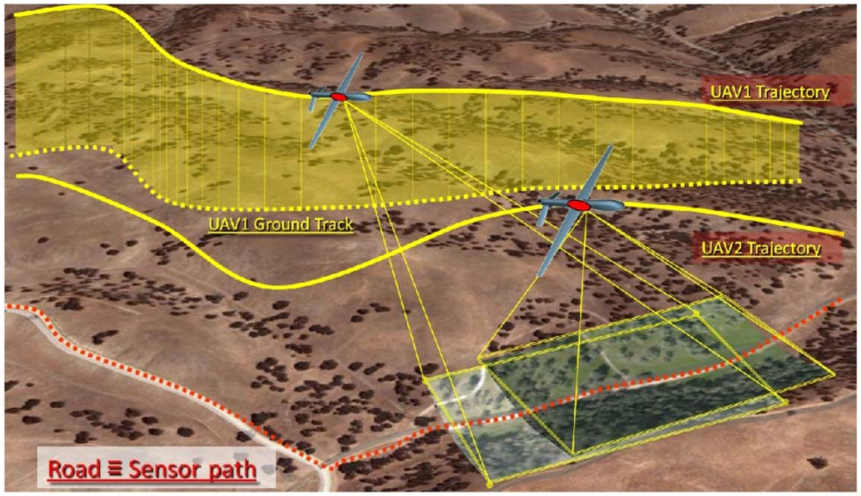
Actuation Period > Actuation Delay

hold, then $\|y_i - y_i^s\|_{\mathcal{L}_\infty} < \gamma_i$.

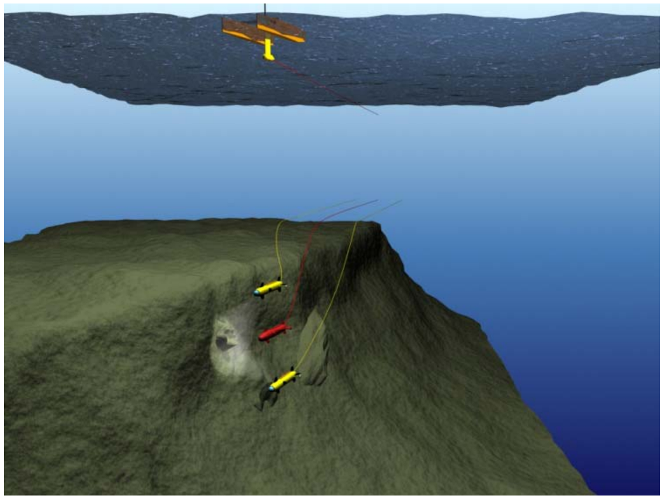
- The performance is subject to the hardware limitations.

New Results on Cooperative Missions

- **Time-critical** applications for multiple vehicles with spatial constraints:
    - Sequential auto-landing (UAVs)
    - Coordinated reconnaissance missions
    - Simultaneous arrival at multiple locations



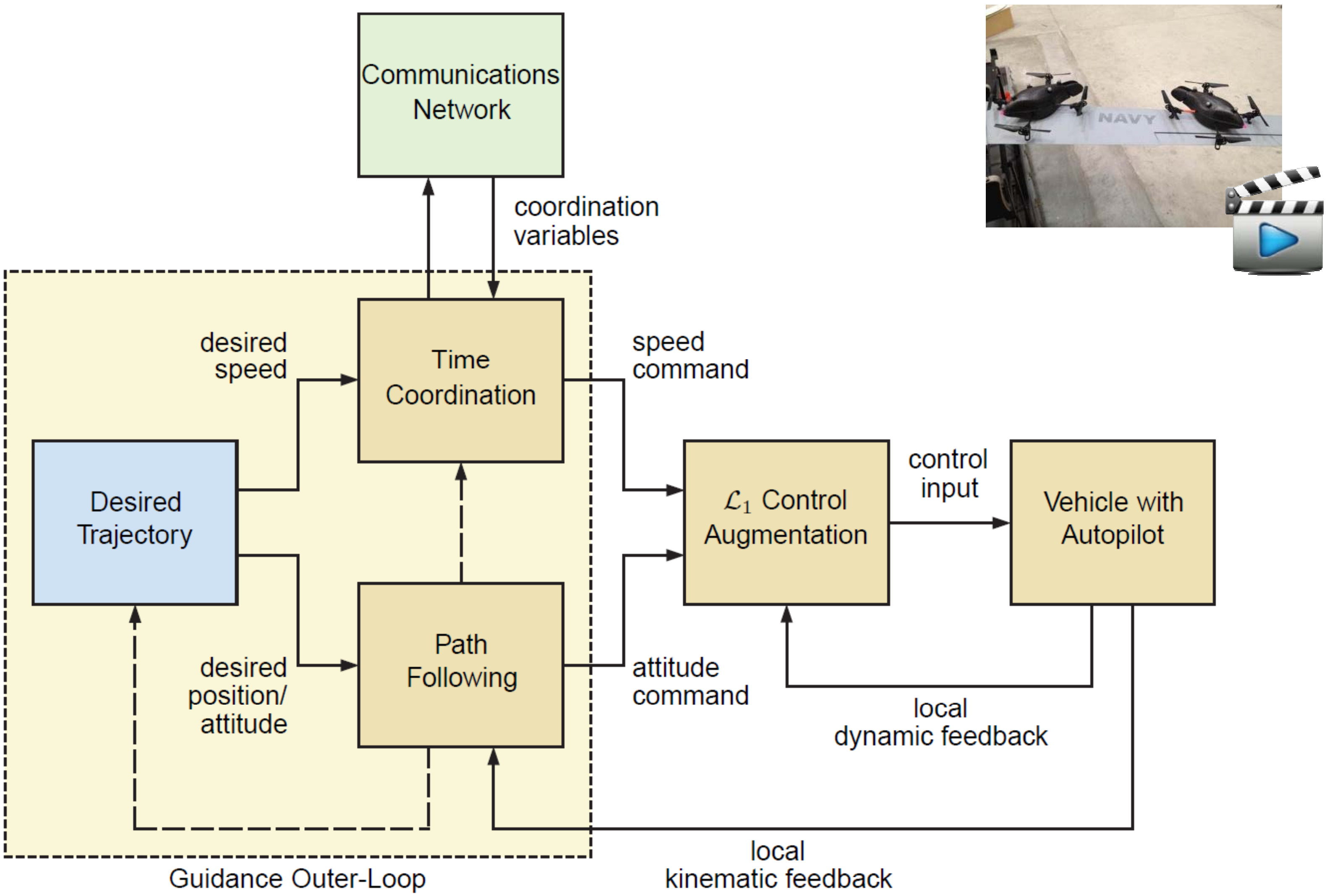
Courtesy: NPS



Courtesy: LARSyS, IST

Coordinate on the **arrival** of a leader (or group of leaders) subject to collision-avoidance, communications, and spatial **constraints**

Overall Conceptual Architecture: Decoupling Space and Time



Xargay, Dobrokhodov, Kaminer, Pascoal, Hovakimyan, & Cao 2012

Time-Critical Coordination :: New Results and Extensions

Consensus problem: reach an *agreement* on some distributed variables of interest *(coordination states)* as well as their rates

$$x_i(t) - x_j(t) \xrightarrow{t \to \infty} 0, \qquad \forall\, i, j \in \mathcal{I}_n$$
$$\dot{x}_i(t) \xrightarrow{t \to \infty} \rho, \qquad \forall\, i \in \mathcal{I}_n$$

Reference rate

Synchronize in both 'position' and 'speed'

- **New results:**
  - New coordination states that accommodate time-varying desired speed profiles;
  - Performance guarantees in the presence of temporary link losses and disconnected graphs:
    - *Lower bound on the (exponential)* ***convergence rate****…*
    - *… as a function of the* ***QoS of the network*** *and the* ***number of leaders****;*
  - In the presence of quantization, existence of equilibrium points corresponding to undesirable steady solutions with zero 'speed':
    - *Derivation of a bound on the* ***quantizer step size*** *to ensure that these equilibria do not exist;*
  - Performance improvement under low connectivity through 'onboard estimators';
  - Performance improvement via emergent leaders.

Key Ideas & Previous Work

- ***PI protocol:***

  Critical to coordinate 'speed';
  Allows agents to learn a reference rate command and reject constant disturbances.

  - *Kaminer, Yakimenko, Pascoal, & Ghabcheloo 2006*
  - *Carli, Chiuso, Schenato, & Zampieri 2008*
  - *Bai, Arcak, & Wen 2008 (generalized PI protocol)*

  Fixed connected graphs;
  No guaranteed convergence rate

- ***Virtual leaders:***

  Allows to track a constant reference rate command with zero steady-state error;
  Drawback: adding agents reduces the connectivity level.

  - *Shi, Wang, & Chu 2006*
  - *Ren & Beard 2007*
  - *Su, Wang, & Lin 2009*

  The 'virtual leader' is implemented as an isolated node providing a reference trajectory

  'Extra agent' imposes a reference rate and its dynamics are affected by other agents

- ***Quantized consensus:***

  Quantization can create equilibrium points corresponding to undesirable steady solutions with zero 'speed'.

  - *Kashyap, Basar, & Srikant 2007*
  - *Censi & Murray 2009*
  - *Nedic, Olshevsky, Ozdaglar, & Tsitsiklis 2009*
  - *Ceragioli, De Persis, & Frasca 2011*

  In the 'conventional' agreement problem, quantization leads to 'practical consensus'

- ***Guaranteed rate of convergence:***

  Critical to ensure successful execution of the mission.

  - *Olfati Saber & Murray 2003*
  - *Kashyap, Basar, & Srikant 2007*
  - *Nedic, Olshevsky, Ozdaglar, & Tsitsiklis 2009*

  Convergence rates derived only for 'conventional' consensus problems

Coordination States :: Time-Varying Speed Profiles

- For **constant desired speed profiles**:
  - ✓ *Normalized curvilinear abscissas*: $\ell'_i(t) \triangleq \dfrac{\ell_i(t)}{\ell_{fi}}$

    $\ell'_i \;:\; [0, t^*_d] \to [0, 1]$

SPACE

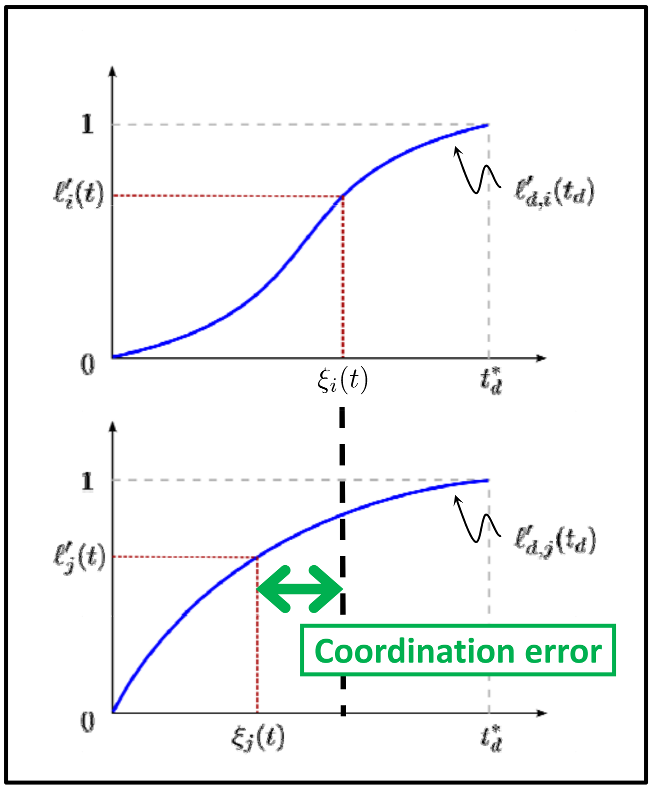
$\rho = v_{d,i}/\ell_{fi}$

- For **time-varying desired speed profiles**:
  - ✓ ***Time** variables*: $\xi_i(t) \triangleq \eta_i(\ell'_i(t))$

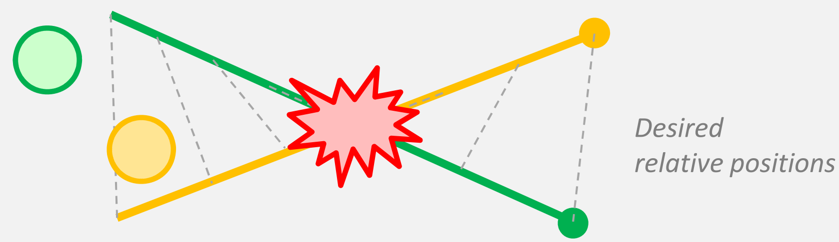
    $\eta_i \;:\; [0, 1] \to [0, t^*_d]$

TIME

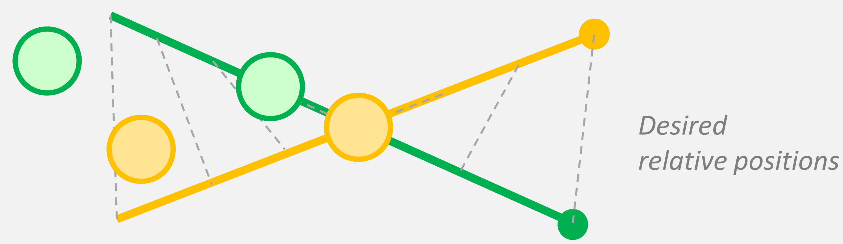
$\rho = 1$



- Normalized abscissas: $\ell'_i(t) - \ell'_j(t)$



- New ***time* variables:** $\xi_i(t) - \xi_j(t)$



❖ *In the case of <u>spatially deconflicted paths</u>, there is no risk of collision; however, **mission-specific goals** will **not** be **satisfied**.*

Agent Dynamics & Communications Network

Consider a **network of *n* integrator-agents**:

$$\dot{x}_i(t) = u_i(t) + d_i, \quad x_i(0) = x_{i0}, \qquad i \in \mathcal{I}_n := \{1, \ldots, n\}$$

with **dynamic information flow** $\mathcal{G}_0(t) := (\mathcal{V}_0, \mathcal{E}_0(t))$.

Communications network:

- Each agent can **only** exchange information with a set of **neighboring agents;**
- Communications between agents are **bidirectional** and information is transmitted **continuously with no delays;**
- **Connectivity** of the communications network at time t satisfies:

$$\frac{1}{n}\frac{1}{T}\int_t^{t+T} \boldsymbol{Q_n L_0}(\tau)\boldsymbol{Q_n^\top} d\tau \geq \mu \mathbf{I}_{\boldsymbol{n-1}}, \quad \forall\, t \geq 0$$

Arcak 2007

- Parameters T and μ characterize the **QoS of the network;**
- Graph connected in an **integral sense**, not piecewise in time:

✓ *The graph may be disconnected during some interval of time…*
✓ *… or **may even fail to be connected at all times**.*

Virtual Agents & Extended Network

- We add n_ℓ **virtual agents** with dynamics:

$$\dot{x}_{\ell i}(t) = u_{\ell i}(t), \quad x_{\ell i}(0) = x_{\ell i0}, \qquad i \in \mathcal{I}_\ell := \{1, \ldots, n_\ell\}$$

Disturbance-free dynamics

❖ To limit the amount of information transmitted over the network, **both leaders and followers only exchange one coordination state with their neighbors**.

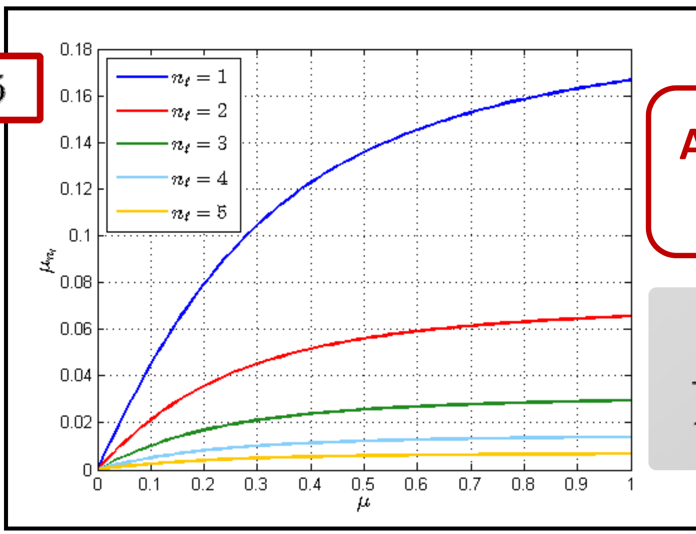
✓ ***Followers*** *exchange their own* ***actual states****.*

✓ ***Leaders*** *only exchange the state of their* ***virtual agents.***

Having multiple leaders improves robustness to single-point failures

Ren & Beard 2007

$n = 5$



Adding virtual agents *reduces* the connectivity level of the extended network

$$\frac{1}{N}\frac{1}{T}\int_{t}^{t+T} \boldsymbol{Q_N L}(\tau)\boldsymbol{Q_N}^\top d\tau \geq \mu_{n_\ell}\mathbb{I}_{\boldsymbol{N-1}}, \quad \forall\, t \geq 0$$

Xargay, Choe, Hovakimyan, & Kaminer 2012

PI Protocol & Collective Dynamics

- **Distributed protocol:**

$$u_{\ell i} = k_P \textstyle\sum_{j \in \mathcal{N}_{\ell i}} (x_j - x_{\ell i}) + \rho, \qquad i \in \mathcal{I}_\ell$$

$$u_i = k_P \textstyle\sum_{j \in \mathcal{N}_i} (x_j - x_i) + \chi_i, \qquad i \in \mathcal{I}_n$$

$$\dot{\chi}_i = k_I \textstyle\sum_{j \in \mathcal{N}_i} (x_j - x_i), \quad \chi_i(0) = \chi_{i0}, \qquad i \in \mathcal{I}_n$$

  - *Reference rate only available to the leaders;*
  - *Proportional-integral control structure:*
    - ✓ Disturbance rejection capabilities;
    - ✓ Followers can learn the reference rate command;
  - *Each vehicle exchanges only its coordination state* $x_\bullet(t)$ *with its neighbors.*

- **Closed-loop collective dynamics:**

Switched LTI system

$$\dot{\boldsymbol{x}}(t) = -k_P \boldsymbol{L}(t)\boldsymbol{x}(t) + \begin{bmatrix} \rho \mathbf{1}_{n_\ell} \\ \boldsymbol{\chi}(t) + \boldsymbol{d} \end{bmatrix}, \qquad \boldsymbol{x}(0) = \boldsymbol{x}_0$$

$$\dot{\boldsymbol{\chi}}(t) = -k_I \boldsymbol{C}^\top \boldsymbol{L}(t)\boldsymbol{x}(t), \qquad \boldsymbol{\chi}(0) = \boldsymbol{\chi}_0$$

Convergence Properties

- Define the **consensus error state** $\zeta(t) := [\zeta_1(t)^\top, \zeta_2(t)^\top]^\top$:

$$\zeta_1(t) := Q_N x(t)$$
$$\zeta_2(t) := \chi(t) - \rho \mathbf{1}_n + d$$

$$\zeta(t) = \mathbf{0} \quad \Leftrightarrow \quad \begin{array}{l} x(t) \in \mathrm{span}\{\mathbf{1}_N\} \\ \dot{x}(t) = \rho \mathbf{1}_N \end{array}$$

- **Closed-loop collective dynamics**:

Switched LTI system

$$\dot{\zeta}(t) = A_\zeta(t)\zeta(t)\,, \qquad \zeta(0) = \zeta_0$$

- There exist **coordination control gains** such that

Proof is constructive

$$\|\zeta(t)\| \leq \alpha_\zeta \|\zeta(0)\|\, \mathrm{e}^{-\lambda_c t}$$

where $$\lambda_c \geq \bar{\lambda}_c := \frac{k_P N \mu_{n_\ell}}{(1+k_P N T)^2}(1+\beta_\lambda)^{-1}\,, \quad \beta_\lambda \geq 2\frac{N}{n_\ell}\sqrt{\frac{N}{n_\ell}}$$

- Moreover, the coordination states and their rates of change satisfy:

$$\lim_{t\to\infty} |x_i(t) - x_j(t)| = 0\,, \qquad i, j \in \mathcal{I}_n$$
$$\lim_{t\to\infty} \dot{x}_i(t) = \rho\,, \qquad i \in \mathcal{I}_n$$

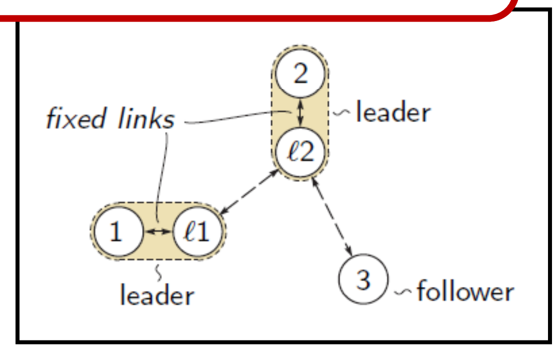
Convergence under Quantization

- PI protocol under **quantized feedback:**

Only information exchanged over the network is quantized

$$\boldsymbol{u} = -k_P \left(\tilde{\boldsymbol{D}}(t)\boldsymbol{x} - \tilde{\boldsymbol{A}}(t)\,\mathrm{q}(\boldsymbol{x}) \right) + \begin{bmatrix} \rho \mathbf{1}_{n_\ell} \\ \boldsymbol{\chi} \end{bmatrix}$$

$$\dot{\boldsymbol{\chi}} = -k_I \boldsymbol{C}^\top \left(\tilde{\boldsymbol{D}}(t)\boldsymbol{x} - \tilde{\boldsymbol{A}}(t)\,\mathrm{q}(\boldsymbol{x}) \right), \quad \boldsymbol{\chi}(0) = \boldsymbol{\chi}_0$$



- Carathéodory solutions **might not exist**; need to consider solutions in the sense of ***Krasovsky***. *(Ceragiolo, De Persis, & Frasca 2011)*

- Potential existence of **undesirable attractors/equilibria** if:

$$\Delta \geq \frac{2n_\ell}{n(n-1)} \frac{|\rho|}{k_P}$$

- **Uniform ultimate boundedness** with ultimate bounds proportional to the step size of the (uniform) quantizers:

$$\begin{aligned} |x_i(t) - x_j(t)| &\leq \alpha_1 \Delta \\ |\dot{x}_i(t) - \rho| &\leq \alpha_2 \Delta \end{aligned}, \qquad \forall\, t \geq T_b$$